

## Estimation of the values and uncertainties of the BIPM Z12-T receiver and antenna delays, for use in differential calibration exercises

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This memorandum presents results and uncertainties for the calibration of hardware delays of the BIPM Z12-T receiver and associated antennas. These are to be used for computing the results and uncertainties of the differential calibration of similar receivers carried out in 2001 and 2002. After recalling some definitions in section 1, section 2 presents the chosen values of the delays of the reference equipment and section 3 discusses the short and long term stability of the hardware delays of a complete system. Section 4 then estimates the uncertainties associated to the results of a differential calibration and the resulting global uncertainty in some time applications.

The main results are summarized in the first two tables below (numbers in bold type) and are to be used for differential calibration in 2001-2002. With additional hypothesis, one can determine the global uncertainty in time applications using P3 measurements, listed in the third table.

BIPM Z12-T system	<b>P1</b>	<b>P2</b>	P1-P2
Delay of the Reference receiver	<b>281.1 ns ± 0.6 ns</b>	<b>295.4 ns ± 0.6 ns</b>	-14.3 ns ± 0.3 ns
Delay of the Reference antenna	<b>24.5 ns ± 2 ns</b>	<b>26.5 ns ± 2 ns</b>	-2.0 ns ± 2 ns
Delay of the Reference system	<b>305.6 ns ± 2.1 ns</b>	<b>321.9 ns ± 2.1 ns</b>	-16.3 ns ± 2.0 ns

Value and uncertainty of hardware delays of the reference system (BIPM Z12-T).

Uncertainty of a calibration result	<b>P1</b>	<b>P2</b>	P1-P2
Receiver only	<b>1.2 ns</b>	<b>1.2 ns</b>	0.3 ns
Total system	<b>2.3 ns</b>	<b>2.3 ns</b>	2.0 ns

Uncertainty to be associated with the results of a differential calibration.

Global uncertainty in ..	P3
Realization of GPS time	4 ns
Time link	3 ns

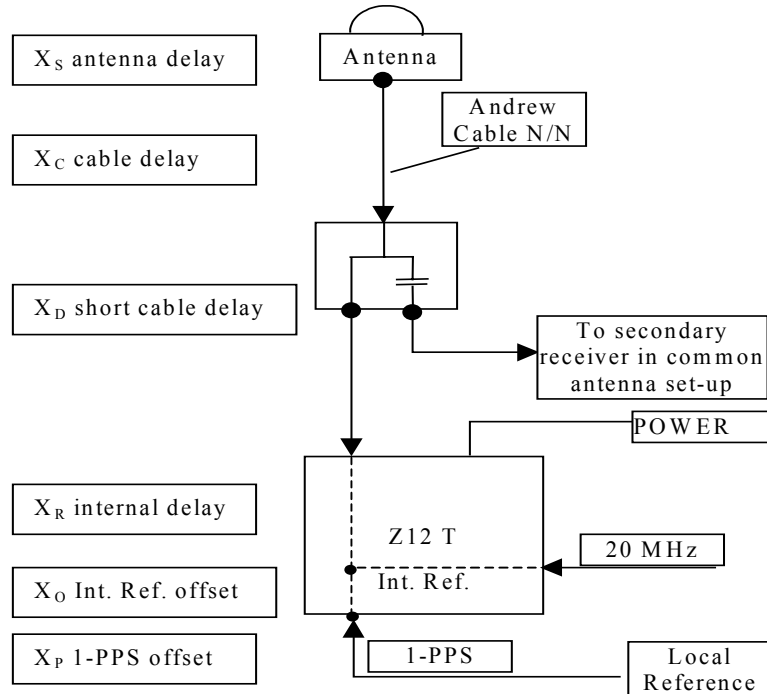
Global uncertainty in time applications using P3 measurements of calibrated receivers

### 1. Definitions

As presented e.g. in (Petit et al., 2001), the calibration of a Z12-T system is divided in (up to) 6 different parts (Figure 1), here listed from the laboratory reference to the antenna:

- $X_P$  = Delay of the 1PPS-in to the laboratory reference
- $X_O$  = Delay of the “internal reference” to the 1PPS-in
- $X_R$  = receiver internal delay, measured from the “internal reference”

- $[X_D = \text{short cable} + \text{splitter}]$  (not present in normal operation)
- $X_C = \text{antenna cable delay}$
- $X_S = \text{antenna delay}$



**Figure 1:** Definition of the different delays used in the typical set-up of the BIPM Z12-T.

The precise definition the “internal reference”, to which the GPS measurements are referred, is also given in (Petit et al. 2001). The values of  $X_P$ ,  $X_O$ ,  $X_D$ ,  $X_C$  are to be measured at each new set-up. The values of  $X_R$  and  $X_S$  are the calibration results.

## 2. Absolute calibration of BIPM Z12-T and associated antennas

We present values of hardware delays at the two GPS frequencies (L1 and L2) for three pieces of equipment: The BIPM Z12-T receiver (S/N LP02944), hereafter named BIPC, (delays  $X_{R1}$  and  $X_{R2}$ ), the 3S Navigation Temperature Stabilized Antenna (S/N 00014), hereafter named TSA, (delays  $X_{T1}$  and  $X_{T2}$ ) and the Ashtech antenna (S/N CR15373) (delays  $X_{S1}$  and  $X_{S2}$ ). The BIPC and the TSA have been absolutely calibrated in May 2000 at the US Naval Research Laboratory (White et al., 2001). For practical purposes, it was later decided to use the Ashtech antenna for the differential calibration exercises, so the transmission delays of this antenna have to be estimated.

- Estimation of  $X_{R1/2}$ : from (White et al., 2001)

$$X_{R1} = 281.1 \text{ ns} \pm 0.6 \text{ ns} \quad X_{R2} = 295.4 \text{ ns} \pm 0.6 \text{ ns} \quad X_{R1} - X_{R2} = -14.3 \text{ ns} \pm 0.3 \text{ ns}$$

- Estimation of  $X_{T1/2}$ : from (White et al., 2001) two sets of values are available, the first one from network analyzer at NRL, the second one from independent measurements at USNO:

$$\begin{array}{ll}
X_{T1} = 30.7 \text{ ns} \pm 2 \text{ ns} & X_{T2} = 23.7 \text{ ns} \pm 2 \text{ ns} \\
X_{T1} = 31.7 \text{ ns} & X_{T2} = 25.3 \text{ ns} \quad (\text{no uncertainty provided})
\end{array}$$

- Estimation of  $X_{S1/2}$ : Four sets of results are available.

Two may be derived from the above measurements of  $X_{Ti}$  using the following result for differential measurements performed at the BIPM:

$$X_{T1} - X_{S1} = 6.5 \text{ ns} \pm 0.2 \text{ ns} \quad X_{T2} - X_{S2} = -2.0 \text{ ns} \pm 0.2 \text{ ns}$$

yielding

$$\begin{array}{ll}
X_{S1} = 24.2 \text{ ns} \pm 2 \text{ ns} & X_{S2} = 25.7 \text{ ns} \pm 2 \text{ ns} \\
X_{S1} = 25.2 \text{ ns} & X_{S2} = 27.3 \text{ ns}
\end{array}$$

Two additional measurements were performed at the NRL in May 2001, using a network analyser and known antennas (E. Powers and J. White, 2001, personal communication)

$$\begin{array}{ll}
X_{S1} = 25 \text{ ns} \pm 1 \text{ ns} & X_{S2} = 28.5 \text{ ns} \pm 1 \text{ ns} \\
X_{S1} = 24.6 \text{ ns} \pm 0.8 \text{ ns} & X_{S2} = 27.1 \text{ ns} \pm 0.8 \text{ ns}
\end{array}$$

Considering the statistical dispersion of these four sets of results (1.0 ns peak to peak for  $X_{S1}$ , 2.8 ns p/p for  $X_{S2}$ , 2.0 ns p/p for  $X_{S2} - X_{S1}$ ) we choose a conventional set to be used for the differential calibration exercise:

$$X_{S1} = 24.5 \text{ ns} \pm 2 \text{ ns} \quad X_{S2} = 26.5 \text{ ns} \pm 2 \text{ ns} \quad X_{S1} - X_{S2} = -2 \text{ ns} \pm 2 \text{ ns}$$

The uncertainty associated to  $X_{S1}$  and  $X_{S2}$  is that of (White et al. 2001). The uncertainty associated to  $X_{S2} - X_{S1}$  is chosen to be the same, implicitly assuming that some correlation exists between the determinations of  $X_{S1}$  and  $X_{S2}$ .

### 3. Study of hardware delay instabilities

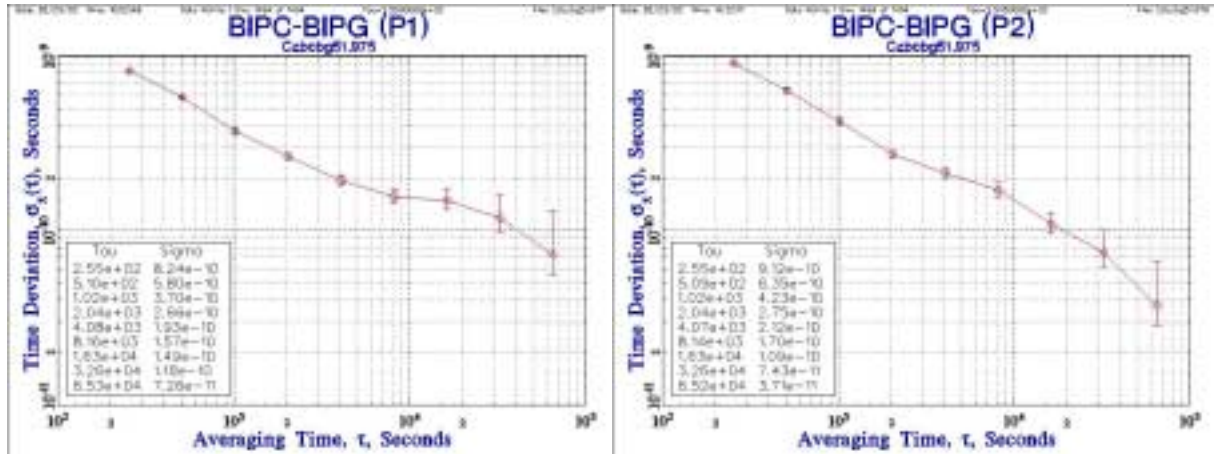
While in section 2 we estimated the uncertainties associated to determinations of the hardware delays, in this section we study the instability of these delays through the instability of the on-site comparison of two different equipments. The short term instability studied in section 3.1 mainly allows to estimate the uncertainty that can be associated to the pseudo-range measurements in a differential calibration. The long term study in section 3.2 gives an hint on the possible variations due to various factors, mostly the manual measurements associated to a calibration exercise and the environment. Note, however, that the two independent systems used are necessarily operated side by side and are subject to similar environmental variations.

#### 3.1 Short term instabilities:

In a differential calibration exercise, pseudo-range measurements are typically taken over a few days and, after analysis, averaged so that a single value represents the difference of hardware delays for each frequency. This is valid if the measurement noise is white phase modulation over the duration of the experiment.

Data taken at the BIPM (March 2001) and at several laboratories during the differential calibration exercise confirm this hypothesis. For example, the Z12-T BIPC has been compared to a Javad Legacy over one week in March 2001. Pseudo-range measurements in the form of 30-s Rinex data are transformed to 13-min REF-GPS measurements in the CCTF format (Defraigne and Petit, 2001) which are differenced between the two receivers. The

differences of 13-min REF-GPS values obtained from both P1 and P2 are characterized by a white phase noise level of about 0.9 ns which averages out to well below 0.1 ns for an averaging duration of one day (Figure 2). Similarly 1-day averages of P1 and P2 measurements are stable at a level of order 0.1 ns during the different days of a typical differential calibration exercise.



**Figure 2:** Time stability of a short-baseline comparison of Z12-T and Javad (P1 and P2)

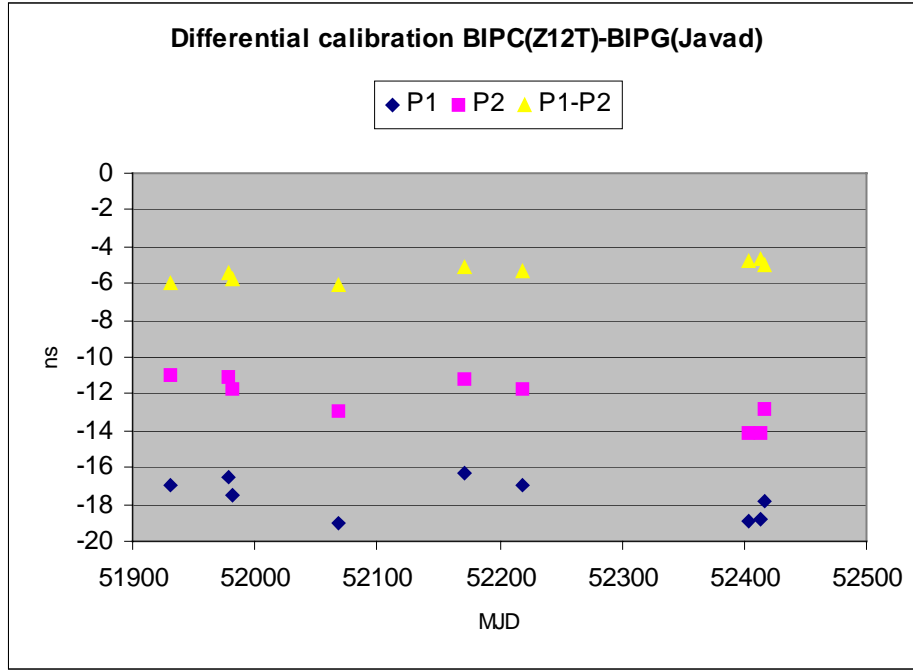
We conclude that the uncertainty from the measurement noise is typically 0.1 ns. However, possible systematic effects due to multipaths cannot be estimated from this test, in which the two antennas were always in the same location.

### 3.2 Long term instabilities:

The BIPM Z12-T and Javad receivers have been compared at 9 occasions over nearly 500 days. The Javad has been kept in a stable set-up over the whole period (except that a 5 MHz input was used, instead of a 10 MHz input, for MJD 52404 only) while the Z12-T has been reset (i.e. completely re-installed with a new measurement of the relative phase of the 20-MHz-in to 1PPS-in signals) 7 times. In addition, 3 different antenna cables, which characteristics may be found below, have been used for the Z12-T over the period.

Cable	Attenuation	Period of use
Andrew FSJ1	14.7 db	51900-52100
LMR 195	28.6 db	52100-52400 and 52416
LMR 400	10.1 db	52401-52415

Because no definition of the reference of the Javad receiver has been chosen yet, it is not possible to interpret the results in terms of a differential calibration of the Javad. Nevertheless the set of results (Figure 3) may be interpreted as the long term instability of the results of a differential calibration, including the instability of (the difference of) the hardware delays of the two receivers, antennas and cables, as well as the repeatability of the measurement of the Z12-T reference ( $X_0$ ) and the uncertainty in the delay measurement for the three different cables used. The standard deviation of the results is 1.0 ns for the P1 differential calibration (2.7 ns peak to peak), 1.2 ns for the P2 differential calibration (3.1 ns p/p) and 0.5 ns for P1-P2 (1.3 ns p/p). It is also to be noted that, although some of this instability may originate from the different attenuations of the cables used, this effect should not be the dominant one because, in the May 2002 (MJD 52400) measurements, an attenuation difference of 18.5 dB resulted in a difference of not more than 1 ns on the differential calibration.



**Figure 3:** Results of several comparisons of the BIPM receivers Z12-T and Javad

We conclude that the long-term instability of hardware delays as well as the repeatability of a differential calibration exercise are at a level of about 1 ns RMS for both P1 and P2, and about 0.5 ns for P1-P2.

#### 4. Uncertainties associated to the results of a differential calibration

For the calibration of a complete system “Study” with respect to a reference system “Ref”, we have

$$(X_{Ri} + X_{Si})(Study) = \Delta P_i + (X_{Ri} + X_{Si} + X_C + X_D - X_0 - X_P)(Ref) - (X_C + X_D - X_0 - X_P)(Study)$$

where  $\Delta P_i$  is the average difference of the pseudo-ranges at frequency  $L_i$ , taking into account the different positions of the antenna phase centers.

Similarly, in a common antenna mode, we have

$$X_{Ri}(Study) = \Delta P_i + (X_{Ri} + X_D - X_0 - X_P)(Ref) - (X_D - X_0 - X_P)(Study)$$

where  $\Delta P_i$  is the measured average difference in pseudo-ranges at frequency  $L_i$ .

The values of  $X_P$  (cable),  $X_O$  (oscilloscope measurement),  $X_C$  and, if necessary,  $X_D$  (cables) are to be measured at each new set-up. Uncertainties are estimated to be of order 0.3 ns for  $X_P$  or  $X_C + X_D$ , and to 0.5 ns for  $X_O$ .

The values of  $X_R$  and  $X_S$  are typically presented as one result per frequency. Because the useful measurements are generally not the pseudo-ranges P1 and P2 at the two frequencies, but rather the ionosphere free pseudo-range  $P_3 = 2.54 \times P_1 - 1.54 \times P_2 = P_1 + 1.54 \times (P_1 - P_2)$ , it may be useful to express the results of calibration (values and uncertainties) as P1 and P1-P2. Particularly  $X_P$ ,  $X_O$ ,  $X_D$  and  $X_C$  have a negligible contribution to the P1-P2 uncertainty.

We can estimate the uncertainty in the results of the differential calibration from the uncertainties of the different parts, as follows:

	L1 or L2	L1-L2
$\Delta P_i$	0.1 ns	0.1 ns
$(X_{Ri})(\text{Ref})$	0.6 ns	0.3 ns
$(X_{Ri} + X_{Si})(\text{Ref})$	2.1 ns	2.0 ns
$(X_c + X_b - X_0 - X_p)(\text{Ref})$	0.7 ns	0.0 ns
$(X_c + X_b - X_0 - X_p)(\text{Study})$	0.7 ns	0.0 ns
We infer the following uncertainties for a differential calibration:		
$(X_{Ri})(\text{Study})$	1.2 ns	0.3 ns
$(X_{Ri} + X_{Si})(\text{Study})$	2.3 ns	2.0 ns

In the following we estimate the effect of this uncertainty on time transfer measurements, but we do not consider measurement noise which, according to e.g. the results in section 3.1, would average to well below 1 ns at one hour and around 0.1 ns at one day. From the table above, the global uncertainty in referring GPS time to a laboratory reference through P3 pseudo-range measurements is of order  $\sqrt{(2.3^2 + (1.54 \times 2.0^2))} \approx 3.8$  ns. Because it is expected that a large part of this uncertainty originates in the absolute calibration procedure itself, and because errors from the absolute calibration of the reference will have the same effects for two systems that have been differentially calibrated with the same reference, it is expected that the uncertainty in a time link computed with two such systems will be lower. For example, assuming (from section 3.2) that the instability of the reference is 1.0 ns for P1 and 0.5 ns for P1-P2, the uncertainty on the ‘non-constant’ part in each calibration would be  $\sqrt{(1^2 + 2 \times (0.7^2))} \approx 1.4$  ns for P1 and 0.5 ns for P1-P2, therefore 1.6 ns for P3 for each system, resulting in an uncertainty of order 2.3 ns for the link.

These results provide indications on the possible accuracy of time transfer with P3 measurements from calibrated receivers. Rounding up to one nanosecond, we estimate that the accuracy in realizing GPS time could be 4 ns and the accuracy of a time link could be 3 ns. We remind that these numbers rely on some assumptions that may be overly optimistic: 1. on the uncertainty in the (L1-L2) antenna delay (end of section 2); 2. on the long-term instability of the receiver and antenna delays (section 3).

## Acknowledgements

Thanks to Philippe Moussay for operating the receivers, performing the necessary measurements and managing the data since the beginning of this experiment, to Zhiheng Jiang for some data processing, and to Peter Wolf for careful reading of the manuscript.

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