FREQUENCY COMPARISON (H\textsubscript{Maser} 40 0890) - (LNE-SYRTE-FO2)
From MJD 54054 to MJD 54069

The primary frequency standard LNE-SYRTE-FO2 was compared to the hydrogen Maser (40 0890) of the laboratory during the 15\textsuperscript{th} to 30\textsuperscript{th} November 2006 period, from MJD 54054 to MJD 54069.

<table>
<thead>
<tr>
<th>Period (MJD)</th>
<th>y(HMaser\textsubscript{40 0890} - FO2)</th>
<th>(u_B)</th>
<th>(u_A)</th>
<th>(u_{\text{link / maser}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>54054 – 54069</td>
<td>-3684.86</td>
<td>3.9</td>
<td>1.8</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Table 1: Results of the comparison in \(1 \times 10^{-16}\).

The measurements of fractional frequency differences are corrected for the systematic frequency shifts listed below.

Technical modifications of SYRTE-FO2 fountain since the last TAI report:

Two main modifications have been made since the last report (February 2006).

- First, all laser beam collimators have been replaced with new dichroïc collimators allowing for the simultaneous operating with caesium and rubidium atoms. This change can slightly modify the atomic cloud geometry and the launch direction which in turn can affect the way the cloud senses the phase distribution in the microwave resonator. After implementing the new collimators, measurements have been done to retune the verticality of the fountain. Measurements are similar to those reported in [1]. The frequency difference measured when feeding the microwave resonator from either side. The difference is made equal to zero by tilting the fountain to ensure that the effective geometry is insensitive to transverse linear phase gradients. Only a minor change of ~200 micro radians was necessary to null the frequency difference with an uncertainty of \(2.2 \times 10^{-16}\). In operation, the cavity is fed symmetrically (percent level in phase and amplitude) to further cancel the linear term. As a consequence, the uncertainty related to residual phase gradient is unchanged compared to previous reports.

- Second, a non-dephasing microwave switch has been implemented on the interrogation source to efficiently reduce microwave leaks. The microwave field is left on only when the atomic cloud is inside the resonator. The switch is appropriately toggled on or off when the cloud is in the cavity cut-offs. As reported in [10], the attenuation of the switch is such that the microwave leaks are reduced below \(10^{-17}\). The phase stability of the field after the switch is turned on has been thoroughly tested has reported in [10]. The potential systematic shift introduced by a putative residual phase transient is estimated to be smaller than \(3 \times 10^{-17}\). Additionally, based on experiments reported in [11], we estimate that other effects related to the microwave synthesis (spurious lines, synchronous phase modulations…) are smaller than \(4 \times 10^{-17}\). Overall, the uncertainty due to microwave synthesis related issues is \(5 \times 10^{-17}\), a significant improvement compared to the corresponding uncertainty previously reported for SYRTE-FO2.
Uncertainties budget of systematic effects in the FO2 fountain

Systematic effects taken into account are the quadratic Zeeman, the Black Body, the cold collision and cavity pulling corresponding to the systematic part, the microwave spectral purity and the microwave leakage, the Ramsey Rabi pulling, the recoil, the 1st and 2nd Doppler and the background collisions. Each of these effects is affected by an uncertainty. The uncertainty of the red shift effect is also included in the systematic uncertainty budget. Systematic uncertainty is estimated by the sum of quadratic systematic uncertainties and gives

\[
\sigma_B = \left( \sigma_{\text{Zeeman}}^2 + \sigma_{\text{BlackBody}}^2 + \sigma_{\text{Collision Syst}}^2 + \sigma_{\text{Microwave Spectrum Leakage}}^2 + \sigma_{\text{first Doppler}}^2 \\
+ \sigma_{\text{Ramsey Rabi}}^2 + \sigma_{\text{Recoil}}^2 + \sigma_{\text{second Doppler}}^2 + \sigma_{\text{Background collisions}}^2 + \sigma_{\text{Redshift}}^2 \right)^{(1/2)}
\]

Table 2 resumes the budget of systematic effects and their associated uncertainties. More details on systematic effects are developed in the next paragraphs.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Correction ((10^{-16}))</th>
<th>Uncertainty ((10^{-16}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic Zeeman effect</td>
<td>-1920.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Black body radiation</td>
<td>168.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Cold collisions and cavity pulling</td>
<td>129.3</td>
<td>1.3</td>
</tr>
<tr>
<td>Microwave spectral purity &amp; leakage</td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>Ramsey &amp; Rabi pulling</td>
<td></td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Microwave recoil</td>
<td></td>
<td>&lt;1.4</td>
</tr>
<tr>
<td>First order Doppler effect</td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>Second order Doppler effect</td>
<td></td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Background gas collisions</td>
<td></td>
<td>&lt;1.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>3.8</strong></td>
</tr>
<tr>
<td>Red shift</td>
<td>-65.4</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>Total with red shift</strong></td>
<td></td>
<td><strong>3.9</strong></td>
</tr>
</tbody>
</table>

Table 2: budget of systematic effects and uncertainties for SYRTE-FO2 fountain

For the November 2006 period it gives

\[
\sigma_B = 3.9 \times 10^{-16}
\]
Measurement of the 2nd order Zeeman frequency shift

Every 20 minutes the frequency of the central fringe of the field linearly dependant transition \([F=3, m_F=1]\) \([F=4, m_F=1]\) is measured. This frequency is directly proportional to the field as \(\delta(v_{11})=K_{Z1}B\) with \(K_{Z1} = 7,0084\ Hz.nT^{-1}\) (see [5] vol. 1 p37 table 1.1.7(a)). In the fountain, the transition \([F=3, m_F=0]\) \([F=4, m_F=0]\) is shifted by quadratic Zeeman effect and depend on squared magnetic field as \(\delta(v_{00})=K_{Z2}B^2\) with \(K_{Z2} = 42,745\ mHz.\mu T^{-2}\) (see [5] vol. 1 p37 table 1.1.7(a)). Knowing \(K_{Z1}\) and measuring \(\delta(v_{11})\) allow good estimation of Zeeman quadratic shift as \(\delta(v_{oo}) = K_{Z2} \left( \frac{\delta(v_{11})}{K_{Z1}} \right)^2\). The relative quadratic Zeeman frequency shift is calculated by \(\delta(v_{oo}) = 427,45 \times 10^{-6} \left( \frac{\delta(v_{11})}{700,84} \right)\) with \(\delta(v_{11})\) in Hz unit and \(v_0 = 9192631770\ Hz\). And the uncertainty is evaluated by \(\frac{\Delta(\delta(v_{oo}))}{v_0} = 427,45 \times 10^{-6} \times 2 \times B \times \Delta(B)\) with \(B\) in mG and \(\Delta(B)\) the standard deviation of the magnetic field. The tracking of the central fringe during MJD 54056 to MJD 54070 shows the good stability of the magnetic field in the interrogation zone (the step of the central fringe of -0,21Hz don’t affect the stability in an average time interval of one day). The frequency variation is taken in the time interval 54056 to 54070, as the standard deviation \(\pm 0.034944957\ Hz\). When taking the standard deviation of variation of the magnetic field \(\Delta(B)\) over the whole period of measurement as the field uncertainty, we find \(4.986\ pT\). The corresponding uncertainty of the correction of the second order Zeeman effect is \(9.4 \times 10^{-18}\). During each period of about 24h of integration an evaluation of the Zeeman effect is calculated assorted with an uncertainty averaged from the tracking of the central fringe during this interval duration of about 24h.

For the central fringe \(M1 = 1424,26975157451\ Hz\), the relative quadratic Zeeman shift is

\[
\frac{\delta(f)_{Zeeman2}}{v_0} = \frac{K_{Z2}M1^2}{K_{Z1}^2v_0}
\]

Frequency quadratic Zeeman shift was evaluated to

\[
\frac{\delta(f)_{Zeeman2}}{v_0} = 1.9204 \times 10^{-13}
\]

The associated uncertainty of the quadratic Zeeman shift was calculated

\[
\sigma_{Zeeman2} = 9.423 \times 10^{-18}
\]

Measurement of the collisional frequency shift and the cavity pulling

Collisional shift takes into account the effect of the collisions between cold Caesium atoms and the effect of "Cavity Pulling" whose influence also depends on the number of atoms. This effect is measured in a differential way during each integration and its determination thus depends on the duration of the measurement and on the stability of the clock, thus the uncertainty on the determination of the collisional shift is mainly of statistical nature. To the statistical uncertainty, we add a type B uncertainty of 1% of frequency shift resulting from the imperfection of the adiabatic passage method (see [4]).
The relative frequency shift due to the effect of the collisions and "Cavity Pulling" of the atomic fountain FO2 were measured in low density, between the MJD 54056 and 54070 with the statistical uncertainty, \( \sigma_{\text{Collision}(i)} \).

The stability of a differential measurement using high and half atom density fountain configurations during MJD 54056 to MJD 54070 using the Allan deviation was calculated, in order to correct of the cold collisional shift for this period. FO2 was operated alternatively (every 50 clock cycles) at low atomic density and high density against the cryogenic oscillator weakly phase locked on the H_Maser890. The measured density ratio between low and high densities is \( 0.50026666 \pm 0.0000620 \).

The frequency difference between both densities is used to determine the collisional coefficient which is used to correct each data point. The Allan deviation varies as \( \tau^{-1/2} \) and reaches \( 10^{-16} \) after 100000s.

The weighted mean of collisional shift gives for November 2006 is

\[
\frac{\sum_{i=1}^{n} \frac{y_{\text{Collision} i}}{\sigma_{\text{Collision} i}^2}}{\sum_{i=1}^{n} \frac{1}{\sigma_{\text{Collision} i}^2}}
\]

\[y_{\text{Collision moy}} := -1.29326 \times 10^{-14}\]

The systematic effect of these shifts is evaluated by the 1\% part of the mean frequency collisional shift during November 2006:

\[
\sigma_{\text{Collision Syst}} = \frac{1}{100} |y_{\text{Collision moy}}| = \left( \sigma_{\text{Collision}} \right)_{\text{Syst}} = 1.2932 \times 10^{-16}
\]

This value is taking into account in the systematic uncertainty evaluation \( \sigma_B \).
3 - Measurement of the Blackbody Radiation shift

An ensemble of 3 platinum thermistors monitors the temperature and its gradient inside the vacuum chamber. The average temperature is \( T \sim 24.8^\circ C \) with a gradient smaller than \( \delta(T) = 0.2 K \) along the atom trajectory. The correction is

\[
\delta(v)_{\text{BlackBody}} = K_{BB} T^4 \left( 1 + \frac{dT}{T_0} \right)
\]

with \( K_{BB} = -1.573 \times 10^{-4} \pm 3 \times 10^{-7} \) \[10\], \( \varepsilon := 0.014\& + 0.0014 \) \[11\]-\[12\], \( T_0 := 300 K \). The Blackbody Radiation shift is assorted of uncertainty obtained with the squared of quadratic sum of \( \delta(K_{BB}), \delta(\varepsilon) \) and \( \delta(T) \):

\[
\frac{\delta(v)}{V_0}_{\text{BlackBody}} = -1.6875 \times 10^{14} \pm 0.6 \times 10^{-16}
\]

4 - Effect of the Microwave Spectrum effect and leakage effect

Microwave leaks are strongly suppressed (smaller than \( 10^{-17} \)) by switching the microwave field off by 40 dB when the atomic cloud is outside the Ramsey cavity. The microwave switch has been specifically developed and tested for this application \[10\]. Systematic effect related to a putative residual phase transient introduced by the switch is estimated to be smaller than \( 3 \times 10^{-17} \). Other effects related to the microwave synthesis have been assessed through phase noise power spectral density measurements, comparisons between synthesizers with strongly different synthesis schemes, \textit{in situ} phase transient analysis, as reported in \[11\]. The corresponding uncertainty is \( 4 \times 10^{-17} \). The overall uncertainty connecting to microwave related issues is \( 5 \times 10^{-17} \).

\[
\frac{\delta(v)}{V_0}_{\text{MicrowaveSpectrum}} = 0 \pm 0.5 \times 10^{-16}
\]

5 - Measurement of the residual 1\textsuperscript{st} order Doppler effect

We determined the frequency shifts caused by asymmetry of the coupling coefficients of the two microwave feedthroughs and the error on the launching direction by coupling the interrogation signal either “from the right” or “from the left” or symmetrically into the cavity. The measured shift is

\[
\frac{\delta(v)}{V_0}_{\text{FirstDoppler}} = 0 \pm 3.0 \times 10^{-16}
\]

In FO2 fountain we feed the cavity symmetrically at 1\% level both in phase and in amplitude. This shift is thus reduced by a factor of 100 and became negligible. The quadratic dependence of the phase becomes dominant. A worse case estimate based on \[6\] gives fractional frequency shift of \( 3 \times 10^{-16} \) which we take as uncertainty due to the residual 1\textsuperscript{st} order Doppler effect.

6 – Rabi and Ramsey effect and Majorana transitions effect

An imbalance between the residual populations and coherences of \( m_F < 0 \) and \( m_F > 0 \) states can lead to a shift of the clock frequency estimated to few \( 10^{-18} \) for a population imbalance of \( 10^{-3} \) that we observe in FO2 (see \[7\] and \[8\]).

7 – Microwave recoil effect

The shift due to the microwave photon recoil was investigated in \[3\]. It is smaller than \( 1.4 \times 10^{-16} \).
8 – Gravitational red-shift and 2nd order Doppler shift

The relativistic effect is evaluated as \( \frac{(\delta (v) \text{ redshift})}{v_0} = \frac{g h}{c^2} \) with \( h = 60 \text{ m} \) \( (\delta (v) \text{ redshift}) \text{ redshift} = 6.540 \times 10^{-15} \)

\( \pm \sigma_{\text{Redshift}} = 0.1 \times 10^{-15} \)

The 2nd order Doppler shift is less than \( 0.08 \times 10^{-16} \).

9 – Background collisions effect

The vacuum pressure inside the fountains is typically a few \( 10^{-8} \) Pa. Based on early measurements of pressure shift (see [5]) the frequency shift due to collisions with the background gas is < \( 10^{-16} \).
REFERENCES


