FREQUENCY COMPARISON (H_MASER 40 0890) - (LNE-SYRTE-FO2)
From MJD 54054 to MJD 54069

The primary frequency standard LNE-SYRTE-FO2 was compared to the hydrogen Maser (40 0890) of the laboratory during the 15th to 30th November 2006 period, from MJD 54054 to MJD 54069.

Period (MJD)	y(HMaser$_{40\ 0890}$ - FO2)	u_B	u_A	$u_{link\ /\ maser}$
54054 – 54069	- 3684,86	3.9	1,8	1,16

Table 1: Results of the comparison in 1 x 10^{-16}.

The measurements of fractional frequency differences are corrected for the systematic frequency shifts listed below.

Technical modifications of SYRTE-FO2 fountain since the last TAI report:

Two main modifications have been made since the last report (February 2006).

- First, all laser beam collimators have been replaced with new dichroïc collimators allowing for the simultaneous operating with caesium and rubidium atoms. This change can slightly modify the atomic cloud geometry and the launch direction which in turn can affect the way the cloud senses the phase distribution in the microwave resonator. After implementing the new collimators, measurements have been done to retune the verticality of the fountain. Measurements are similar to those reported in [1]. The frequency difference measured when feeding the microwave resonator from either side. The difference is made equal to zero by tilting the fountain to ensure that the effective geometry in insensitive to transverse linear phase gradients. Only a minor change of ~200 micro radians was necessary to null the frequency difference with an uncertainty of 2.2×10^{-16}. In operation, the cavity is fed symmetrically (percent level in phase and amplitude) to further cancel the linear term. As a consequence, the uncertainty related to residual phase gradient is unchanged compared to previous reports.
- Second, a non-dephasing microwave switch has been implemented on the interrogation source to efficiently reduce microwave leaks. The microwave field is left on only when the atomic cloud is inside the resonator. The switch is appropriately toggled on or off when the cloud is in the cavity cut-offs. As reported in [10], the attenuation of the switch is such that the microwave leaks are reduced below 10^{-17}. The phase stability of the field after the switch is turned on has been thoroughly tested has reported in [10]. The potential systematic shift introduced by a putative residual phase transient is estimated to be smaller than 3×10^{-17}. Additionally, based on experiments reported in [11], we estimate that other effects related to the microwave synthesis (spurious lines, synchronous phase modulations…) are smaller than 4×10^{-17}. Overall, the uncertainty due to microwave synthesis related issues is 5×10^{-17}, a significant improvement compared to the corresponding uncertainty previously reported for SYRTE-FO2.

Uncertainties budget of systematic effects in the FO2 fountain

Systematic effects taken into account are the quadratic Zeeman, the Black Body, the cold collision and cavity pulling corresponding to the systematic part, the microwave spectral purity and the microwave leakage, the Ramsey Rabi pulling, the recoil, the 1st and 2nd Doppler and the background collisions. Each of these effects is affected by an uncertainty. The uncertainty of the red shift effect is also included in the systematic uncertainty budget. Systematic uncertainty is estimated by the sum of quadratic systematic uncertainties and gives

$$\sigma_B = \left(\sigma_{Zeeman2}{}^2 + \sigma_{BlackBody}{}^2 + \sigma_{Collision_{Syst}}{}^2 + \sigma_{Microwave\_Spectrum\_Leakage}{}^2 + \sigma_{first\_Doppler}{}^2 + \sigma_{Ramsey\_Rabi}{}^2 + \sigma_{Recoil}{}^2 + \sigma_{second\_Doppler}{}^2 + \sigma_{Background\_collisions}{}^2 + \sigma_{Redshift}{}^2 \right)^{(1/2)}$$

Table 2 resumes the budget of systematic effects and their associated uncertainties. More details on systematic effects are developed in the next paragraphs.

	Correction (10^{-16})	**Uncertainty (10^{-16})**
Quadratic Zeeman effect	- 1920.4	0.1
Black body radiation	168.7	0.6
Cold collisions and cavity pulling	129.3	1.3
Microwave spectral purity & leakage		0.5
Ramsey & Rabi pulling		< 0.1
Microwave recoil		< 1.4
First order Doppler effect		3.0
Second order Doppler effect		< 0.1
Background gas collisions		<1.0
Total		**3.8**
Red shift	- 65.4	1.0
Total with red shift		**3.9**

Table 2: budget of systematic effects and uncertainties for SYRTE-FO2 fountain

For the November 2006 period it gives

$$\sigma_B = 3.9 \quad 10^{-16}$$

1 - Measurement of the 2nd order Zeeman frequency shift

Every 20 minutes the frequency of the central fringe of the field linearly dependant transition |F=3, m_F=1⟩ |F=4, m_F=1⟩ is measured. This frequency is directly proportional to the field as $\delta(\nu_{11})=K_{Z1}B$ with K_{Z1} = 7,0084 Hz.nT^{-1} (see **[5]** vol. 1 p37 table 1.1.7(a)). In the fountain, the transition |F=3, m_F=0⟩ |F=4, m_F=0⟩ is shifted by quadratic Zeeman effect and depend on squared magnetic field as $\delta(\nu_{00})=K_{Z2}B^2$ with K_{Z2} = 42,745 mHz.µT^{-2} (see **[5]** vol. 1 p37 table 1.1.7(a)). Knowing K_{Z1} and measuring δ(ν11) allow good estimation of Zeeman quadratic shift as $\delta(\nu_{00})=K_{Z2}\left(\frac{\delta(\nu_{11})}{K_{Z1}}\right)^2$. The relative quadratic Zeeman frequency shift is calculated by $\frac{\delta(\nu_{00})}{\nu_0}=\frac{427{,}45\times10^{-6}\left(\frac{\delta(\nu_{11})}{700{,}84}\right)}{\nu_0}$ with $\delta(\nu_{11})$ in Hz unit and ν_0 = 9192631770 Hz. And the uncertainty is evaluated by $\frac{\Delta(\delta(\nu_{00}))}{\nu_0}=427{,}45\times10^{-6}\times\frac{2\times\bar{B}\times\Delta(B)}{\nu_0}$ with B in mG and $\Delta(B)$ the standard deviation of the magnetic field. The tracking of the central fringe during MJD 54056 to MJD 54070 shows the good stability of the magnetic field in the interrogation zone (the step of the central fringe of -0,21Hz don't affect the stability in an average time interval of one day). The frequency variation is taken in the time interval 54056 to 54070, as the standard deviation ± **0.034944957 Hz**. When taking the standard deviation of variation of the magnetic field $\Delta(B)$ over the whole period of measurement as the field uncertainty, we find **4.986 pT**. The corresponding uncertainty of the correction of the second order Zeeman effect is **9.4 x 10^{-18}**. During each period of about 24h of integration an evaluation of the Zeeman effect is calculated assorted with an uncertainty averaged from the tracking of the central fringe during this interval duration of about 24h.

For the central fringe *M1 = 1424,26975157451 Hz* , the relative quadratic Zeeman shift is

$$\frac{\delta(f)_{Zeeman2}}{\nu_0}=\frac{K_{Z_2}M1^2}{K_{Z_1}^2\nu_0}$$

Frequency quadratic Zeeman shift was evaluated to

$$\frac{\delta(f)_{Zeeman2}}{\nu_0}=1.9204\text{ x }10^{-13}$$

The associated uncertainty of the quadratic Zeeman shift was calculated

$$\sigma_{Zeeman2}=9.423\ 10^{-18}$$

2 - Measurement of the collisional frequency shift and the cavity pulling

Collisional shift takes into account the effect of the collisions between cold Caesium atoms and the effect of "Cavity Pulling" whose influence also depends on the number of atoms. This effect is measured in a differential way during each integration and its determination thus depends on the duration of the measurement and on the stability of the clock, thus the uncertainty on the determination of the collisional shift is mainly of statistical nature. To the statistical uncertainty, we add a type B uncertainty of 1% of frequency shift resulting from the imperfection of the adiabatic passage method (see **[4]**).

The relative frequency shift due to the effect of the collisions and "Cavity Pulling" of the atomic fountain FO2 were measured in low density, between the MJD 54056 and 54070 with the statistical uncertainty, $\sigma_{Collision(i)}$.

The stability of a differential measurement using high and half atom density fountain configurations during MJD 54056 to MJD 54070 using the Allan deviation was calculated, in order to correct of the cold collisional shift for this period. FO2 was operated alternatively (every 50 clock cycles) at low atomic density and high density against the cryogenic oscillator weakly phase locked on the H_Maser890. The measured density ratio between low and high densities is **0,50026666 ± 0,0000620**. The frequency difference between both densities is used to determine the collisional coefficient which is used to correct each data point. The Allan deviation varies as $\tau^{-1/2}$and reaches 10^{-16} after 100000s.

The weighted mean $y_{Collision_{moy}} = \dfrac{\sum_{i=1}^{n} \dfrac{y_{Collision_i}}{\sigma_{Collision_i}^2}}{\sum_{i=1}^{n} \dfrac{1}{\sigma_{Collision_i}^2}}$ of collisionnal shift gives for November 2006 is

$$y_{Collision_{moy}} := -1.29326\ 10^{-14}$$

The systematic effect of these shifts is evaluated by the 1% part of the mean frequency collisional shift during November 2006:

$$\sigma_{Collision_{Syst}} = \frac{1}{100}\left|y_{Collision_{moy}}\right| = \boxed{(\sigma_{Collision})_{Syst} = 1.2932\ 10^{-16}}$$

This value is taking into account in the systematic uncertainty evaluation σ_B .

3 - Measurement of the Blackbody Radiation shift

An ensemble of 3 platinum thermistors monitors the temperature and its gradient inside the vacuum chamber. The average temperature is T~24,8°C with a gradient smaller than $\delta(T) = 0.2\,K$ along the atom trajectory. The correction is

$$\left(\frac{\delta(\nu)}{\nu_0}\right)_{BlackBody} = \frac{K_{BB} T^4 \left(1 + \frac{\varepsilon T^2}{T_0^2}\right)}{T_0^4}$$

with $K_{BB} = -1.573 \times 10^{-4} \pm 3 \times 10^{-7}$ **[10]**, $\varepsilon := 0.014 \& + -0.0014$ **[11]**,**[12]**, $T_0 := 300\,K$. The Blackbody Radiation shift is assorted of uncertainty obtained with the squared of quadratic sum of $\delta(K_{BB})$, $\delta(\varepsilon)$ and $\delta(T)$:

$$\left(\frac{\delta(\nu)}{\nu_0}\right)_{BlackBody} = -1.6875 \times 10^{14} \pm 0.6 \times 10^{-16}$$

4 - Effect of the Microwave Spectrum effect and leakage effect

Microwave leaks are strongly suppressed (smaller than 10^{-17}) by switching the microwave field off by 40 dB when the atomic cloud is outside the Ramsey cavity. The microwave switch has been specifically developed and tested for this application **[10]**. Systematic effect related to a putative residual phase transient introduced by the switch is estimated to be smaller than 3×10^{-17}. Other effects related to the microwave synthesis have been assessed through phase noise power spectral density measurements, comparisons between synthesizers with strongly different synthesis schemes, *in situ* phase transient analysis, as reported in **[11]**. The corresponding uncertainty is 4×10^{-17}. The overall uncertainty connecting to microwave related issues is 5×10^{-17}.

$$\left(\frac{\delta(\nu)}{\nu_0}\right)_{MicrowaveSpectrum} = 0 \pm 0.5 \times 10^{-16}$$

5 - Measurement of the residual 1st order Doppler effect

We determined the frequency shifts caused by asymmetry of the coupling coefficients of the two microwave feedthroughs and the error on the launching direction by coupling the interrogation signal either “from the right” or “from the left” or symmetrically into the cavity. The measured shift is

$$\left(\frac{\delta(\nu)}{\nu_0}\right)_{FirstDoppler} = 0 \pm 3.0 \times 10^{-16}$$

In FO2 fountain we feed the cavity symmetrically at 1% level both in phase and in amplitude. This shift is thus reduced by a factor of 100 and became negligible. The quadratic dependence of the phase becomes dominant. A worse case estimate based on **[6]** gives fractional frequency shift of 3×10^{-16} which we take as uncertainty due to the residual 1st order Doppler effect.

6 – Rabi and Ramsey effect and Majorana transitions effect

An imbalance between the residual populations and coherences of $m_F < 0$ and $m_F > 0$ states can lead to a shift of the clock frequency estimated to few 10^{-18} for a population imbalance of 10^{-3} that we observe in FO2 (see **[7]** and **[8]**).

7 – Microwave recoil effect

The shift due to the microwave photon recoil was investigated in **[3]**. It is smaller than $1{,}4 \times 10^{-16}$.

8 – Gravitational red-shift and 2nd order Doppler shift

The relativistic effect is evaluated as $\frac{(\delta(\nu))_{redshift}}{\nu_0} = \frac{g\,h}{c^2}$ with h=60m $\frac{(\delta(\nu))_{redshift}}{\nu_0} = 6.540\ 10^{-15}$ $\pm\ \sigma_{Redshift} = 0.1\ \ 10^{-15}$

The 2nd order Doppler shift is less than 0,08 x 10^{-16} .

9 – Background collisions effect

The vacuum pressure inside the fountains is typically a few 10^{-8} Pa. Based on early measurements of pressure shift (see **[5]**) the frequency shift due to collisions with the background gas is < 10^{-16}.

REFERENCES

[1] - C. Vian, P. Rosenbusch, H. Marion, S. Bize, L. Cacciapuoti, S. Zhang, M. Abgrall, D. Chambon, I. Maksimovic, P. Laurent, G. Santarelli, A. Clairon of Obs. Paris, SYRTE, A. Luiten, M. Tobar, Univ. W. of Australia School of Physics, C. Salomon of LKB, "LNE-SYRTE Fountains: Recent Results", **CPEM 2004, Proceedings IEEE Transactions on Instrumentation & Measurement, Vol. 54, N0. 2, April 2005.**

[2] - F. Pereira Do Santos, H. Marion, M. Abgrall, S. Zhang, Y. Sortais, S. Bize, I. Maksimovic, D. Calonico, J. Grünert, C. Mandache, C. Vian, P. Rosenbusch, P. Lemonde, G. Santarelli, Ph. Laurent and A. Clairon of LNE-SYRTE, C. Salomon of LKB, "Rb and Cs Laser Cooled Clocks: Testing the Stability of Fundamental Constants". **Proceedings IEEE 2003, EFTF Tampa May 2003, p 55-67.**

[3] - P. Wolf of LNE SYRTE, C.J. Bordé of LPL, "Recoil effects in microwave Ramsey spectroscopy", arxiv: **quant-ph/0403194**.

[4] - F. Pereira Do Santos, H. Marion, S. Bize, Y. Sortais, A. Clairon, and C. Solomon "Controlling the Cold Collision Shift in High Precision Atomic Interferometry" of, **Phys, Rev, Lett, 89,233004 (2002)**.

[5] - J. Vanier, C. Audouin, « The Quantum Physics of Atomic Frequency Standards », **Adam Hilger, Bristol & Philadelphia (1989)**.

[6] - R. Schröder, U. Hübner and D. Griebsch, "Design and Realization of the Microwava Cavity in PTB Caesium Atomic Clock CSF1" **IEEE Trans. Instrum. Meas., vol. 49, p.383, 2002**.

[7] - L. Cutler *et al*, "Frequency pulling by hyperfine σ - transitions in cesium atomic frequency standards, **J. Appl. Phys. Vol. 69, pp. 2780, 1991**.

[8] - A. Bauch, R. Schröder "Frequency shift in Caesium Atomic Clock due to Majorana transitions", Annalen der Physik**, vol. 2, pp. 421, 1993**.

[9] – H. Marion thèse de doctorat de l'Université de Paris 6 (Mars 2005)

[10] - G. Santarelli *et al.*, Proceedings of the 2006 EFTF (2006).

[11] - D. Chambon *et al.*, IEEE Trans. Ultrason. Ferroelect. Freq. Contr., *in press*.