Evaluation of the frequency of the H-maser 1401708 by the primary frequency standard NPL-CsF2

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I. Summary

The primary frequency standard NPL-CsF2 was used to measure the frequency of the H-maser HM2 identified by the clock code 1401708 during seven evaluation campaigns between March and November 2009. These evaluations were performed by measuring mean frequency differences over the reporting periods. The frequency differences are given in table 1, together with the total uncertainty in relating NPL-CsF2 to maser 1401708.

NPL-CsF2 is a new primary frequency standard and the evaluations reported here demonstrate its long-term behaviour and stability. The report presents a brief description of the measurement procedure and a discussion of the uncertainties. A detailed description of the measurement procedure together with a complete evaluation of the systematic frequency biases and their uncertainties is given in reference [1].

Period	MJD	$y(CsF2 - HM2)$ [$\times10^{-15}$]	$u(total)$ [$\times10^{-15}$]
1.	54904 – 54934	4.04	0.58
2.	54974 – 54984	6.79	0.80
3.	55004 – 55014	–1.51	1.30
4.	55039 – 55049	–1.56	0.90
5.	55064 – 55074	3.60	0.93
6.	55084 – 55114	5.98	0.56
7.	55119 – 55144	9.56	0.59

Table 1. Summary of the frequency measurements of clock 1401708 (HM2) including the total uncertainties.

II. Measurement methods

Systematic frequency biases

A number of systematic effects were identified for NPL-CsF2. The complete list of those effects is given in the attached reference [1] together with a detailed description of the methodology for their evaluations.
NPL-CsF2 was corrected for frequency shifts due to the following effects:

- second order Zeeman effect,
- blackbody radiation,
- cold collisions (spin-exchange) and
- gravitational shift.

Other systematic biases were considered small and not corrected for; their uncertainties were included in the total uncertainty budget.
The fountain was mostly operated in the vicinity of the collisional shift cancellation point [2]. The residual collisional shift was constantly measured and corrected for by alternating between high and low atom density; the correct frequency was found by extrapolating the measured frequency values to zero-density. The statistical uncertainty of the extrapolated frequency automatically included the type A uncertainty of the collisional shift. A type B uncertainty for this shift was also identified. This is due to the uncertainty of the high-to-low density ratio, as only the atom number ratio was measured. A value of 10% of the average collisional frequency shift was assigned for the type B uncertainty of this shift.

Measurement procedure

The frequency of NPL-CsF1 was related to a hydrogen maser HM2. The primary standard and the maser were located within the same building and a cable (laid entirely in temperature controlled environment) maintained a 100 MHz link between them.

During the measurement campaign the fountain was run exclusively in the alternating mode between high and low atom number. The ratio of the atom number (high-to-low) was maintained at $\kappa \approx 2$. The fountain’s frequency was continuously extrapolated to the zero density value; therefore, long-term stability of κ was not required. The short-term stability for both high and low atom number was limited by the local oscillator noise, thus the same averaging time at each density level was used.

Corrections due to effects other than collisions were sufficiently stable over the campaign periods, so that single correction values were applied. Nevertheless, the validity of these values, where possible, was checked during a campaign. The C-field magnitude was verified approximately every week. The temperature of the flight tube was measured at one-minute intervals and recorded as two-hour averages.

Uncertainties of the measurement

Short-term stability and type A uncertainty

The short-term stability of the measurement of the frequency extrapolated to zero atomic density was typically 5.4×10^{-13} in 1s. The type A uncertainty of the complete measurement u_A was obtained by assuming white FM noise over the effective period of integration (effective period = reporting period × duty cycle).

Type B uncertainty

The NPL-CsF2 frequency is corrected for the following four effects: second order Zeeman shift, blackbody radiation, cold collisions (spin exchange) and gravitational shift. Except for the gravitational shift, the actual values of the frequency offsets are specific for particular measurement campaigns. In a table below, we reproduce from [1] a list of the identified systematic effects and their uncertainties. In case of the frequency shift due to cold collisions, its uncertainty is a fixed fraction of the shift [1]. Because this uncertainty takes specific values for particular campaigns, it is not included in the table; it is included in total u_B values for the relevant campaigns in chapter III.

Type B evaluation	Uncertainty / 10^{-16}
Second order Zeeman	0.8
Blackbody radiation	1.1
AC Stark (lasers)	0.1
Microwave spectrum	0.1
Gravity	0.5
Collisions with background gas	1.0
Rabi, Ramsey pulling	0.1
Cavity phase (distributed)	3.0
Cavity phase (dynamic)	0.1
Cavity pulling	0.2
Microwave leakage	1.0
Microwave recoil	1.5
Second-order Doppler	0.1
Total (excluding cold collisions)	**4.0**

Link with the local time scale

The uncertainty of the link with the local time scale $u_{l/lab}$ is in general a quadratic sum of two contributions:

$$(u_{l/lab})^2 = (u_{link})^2 + (u_{dt})^2$$

where u_{link} is the uncertainty associated with the frequency transfer between NPL-CsF2 and HM2, and u_{dt} is an additional uncertainty of the measured maser frequency due to gaps (dead time) in the operation of the fountain standard.

In our current arrangement the u_{link}, which normally arises from instabilities of the temperature of the linking cable, is estimated below 10^{-17} and is neglected, giving:

$$(u_{l/lab})^2 = (u_{dt})^2$$

Dead time

During the evaluation period there were gaps in the data collection (dead time) due to both intentional and unintentional breaks. Intentional breaks were required for system maintenance and checks. The unintentional breaks were infrequent and did not have a prevailing cause. In some cases additional dead time was assumed when the measurement period did not match the BIPM reporting times (set every 5 days).

The uncertainty introduced by the dead time, u_{dt}, was approximated by calculating the time deviation (TDEV) of each gap, using the relation between TDEV and the modified Allan deviation (MDEV):

$$\sigma_x^{\,2}(\tau) = \tau^2/3 \;\; mod\sigma_y^{\,2}(\tau)\,,$$

where $\sigma_x(\tau)$ is the TDEV and $mod\sigma_y(\tau)$ is the MDEV.

The stability of the maser 1401708 has been shown to be dominated by flicker frequency modulation (ref. [3]) and the Hadamard deviation has been measured to be

$$\sigma_h(\tau) \cong 1.2 \times 10^{-15};\;\; 500 < \tau[\mathrm{s}] < 6 \times 10^5.$$

The approximation, $mod\sigma_y(\tau) = \sigma_h(\tau)$, was used.

The fractional frequency uncertainty, u_{dt}, arising from the dead time was approximated by the square root of the sum of the time variances, normalised by the length of the measurement campaign, thus:

$$u_{dt} = \frac{1}{T}\sqrt{\sum_{i=1}^{N}\left[\sigma_x(\tau_i)\right]^2}$$

where $\sigma_x(\tau_i)$ is the time deviation (TDEV) of the maser over a duration τ_i; and T is the duration of the campaign.

The uncertainty due to an uncorrected frequency drift of the maser 1401708 for the longest gap is known (ref. [3]) to be negligible when compared to the type A uncertainty of NPL-CsF2 over the same duration. The frequency drift of the maser was therefore omitted from the total uncertainty.

References:

[1] K. Szymaniec, S.E. Park, G. Marra and W. Chalupczak, *Metrologia*, **47**, pp. 363-376 (2010).

[2] K. Szymaniec, W. Chalupczak, E. Tiesinga, C.J. Williams, S. Weyers, R. Wynands, *Phys. Rev. Lett.* **98**, 153002 (2007).

[3] J.A. Davis, C.A. Greenhall, P.W. Stacey, *Metrologia*, **42**, pp. 1-10 (2005).

III. Results of the evaluation

Results of the frequency measurements for the seven campaign periods are listed in the tables below. Frequency biases are given for information only. The given fractional frequency difference $y(CsF2 - HM2)$ is a value corrected for those biases. Note that the values for the collisional shift and its uncertainty vary, and so vary the total type B uncertainties u_B for particular campaigns. The total uncertainty u_{total} of the measurement is defined as:

$$(u_{total})^2 = (u_A)^2 + (u_B)^2 + (u_{l/lab})^2$$

Campaign 1.

Period	MJD	54904 – 54934
Duration	days	30
duty cycle	%	93.8
Biases: 2nd order Zeeman BBR shift cold collisions gravity	$\times 10^{-15}$	 337.15 –16.63 0.66 1.30
y(CsF2 - HM2)	$\times 10^{-15}$	**4.04**
u_A	$\times 10^{-15}$	0.41
u_B	$\times 10^{-15}$	0.41
$u_{l/lab}$	$\times 10^{-15}$	0.07
$\boldsymbol{u_{total}}$	$\times 10^{-15}$	**0.58**

Campaign 2.

Period	MJD	54974 – 54984
duration	days	10
duty cycle	%	93.3
Biases: 2^{nd} order Zeeman BBR shift cold collisions gravity	$\times10^{-15}$	 337.30 –16.65 –2.12 1.30
y(CsF2 - HM2)	$\times10^{-15}$	**6.79**
u_A	$\times10^{-15}$	0.66
u_B	$\times10^{-15}$	0.45
$u_{l/lab}$	$\times10^{-15}$	0.05
$\boldsymbol{u_{total}}$	$\times10^{-15}$	**0.80**

Campaign 3.

Period	MJD	55004 – 55014
duration	days	10
duty cycle	%	64.0
Biases: 2^{nd} order Zeeman BBR shift cold collisions gravity	$\times10^{-15}$	 337.30 –16.54 4.43 1.30
y(CsF2 - HM2)	$\times10^{-15}$	**–1.51**
u_A	$\times10^{-15}$	1.08
u_B	$\times10^{-15}$	0.59
$u_{l/lab}$	$\times10^{-15}$	0.43
$\boldsymbol{u_{total}}$	$\times10^{-15}$	**1.30**

Campaign 4.

Period	MJD	55039 – 55049
duration	days	10
duty cycle	%	76.4
Biases:	$\times 10^{-15}$	
2^{nd} order Zeeman		337.81
BBR shift		–16.54
cold collisions		–1.68
gravity		1.30
y(CsF2 - HM2)	$\times 10^{-15}$	**–1.56**
u_A	$\times 10^{-15}$	0.74
u_B	$\times 10^{-15}$	0.43
$u_{l/lab}$	$\times 10^{-15}$	0.28
$\boldsymbol{u_{total}}$	$\times 10^{-15}$	**0.90**

Campaign 5.

Period	MJD	55064 – 55074
Duration	days	10
duty cycle	%	87.7
Biases:	$\times 10^{-15}$	
2^{nd} order Zeeman		337.81
BBR shift		–16.54
cold collisions		–3.27
gravity		1.30
y(CsF2 - HM2)	$\times 10^{-15}$	**3.60**
u_A	$\times 10^{-15}$	0.76
u_B	$\times 10^{-15}$	0.51
$u_{l/lab}$	$\times 10^{-15}$	0.15
$\boldsymbol{u_{total}}$	$\times 10^{-15}$	**0.93**

Campaign 6.

Period	MJD	55084 – 55114
duration	days	30
duty cycle	%	91.3
Biases: 2nd order Zeeman BBR shift cold collisions gravity	$\times10^{-15}$	 337.81 –16.54 –1.21 1.30
y(CsF2 - HM2)	$\times10^{-15}$	**5.98**
u_A	$\times10^{-15}$	0.38
u_B	$\times10^{-15}$	0.41
$u_{l/lab}$	$\times10^{-15}$	0.07
$\boldsymbol{u_{total}}$	$\times10^{-15}$	**0.56**

Campaign 7.

Period	MJD	55119 – 55144
Duration	days	25
duty cycle	%	90.6
Biases: 2nd order Zeeman BBR shift cold collisions gravity	$\times10^{-15}$	 337.81 –16.54 –1.10 1.30
y(CsF2 - HM2)	$\times10^{-15}$	**9.56**
u_A	$\times10^{-15}$	0.42
u_B	$\times10^{-15}$	0.41
$u_{l/lab}$	$\times10^{-15}$	0.08
$\boldsymbol{u_{total}}$	$\times10^{-15}$	**0.59**