

## Evaluation of $\overline{y(\text{UTC}(\text{NICT}))}$ with respect to NICT-Sr1 for the period MJD 59399 to 59424

The frequency of hydrogen maser HM<sub>1402012</sub> was evaluated for the period from MJD 59399 to 59424 (July 4 – July 29 in 2021) using secondary frequency standard NICT-Sr1. In terms of the fractional deviation from the nominal frequency, we find  $\overline{y(\text{HM}_{1402012})} = -1.822\,646 \times 10^{-12}$ .

We then use the measurements of the dual-mixer time-difference (DMTD) system continuously operated as part of NICT's Japan Standard Time generation to determine the frequency deviation of the time scale UTC(NICT) to be  $\overline{y(\text{UTC}(\text{NICT}))} = -4.270 \times 10^{-15}$ , as determined by NICT-Sr1.

The optical lattice clock acquired data for 24 188 s (1.1% of the total evaluation period) over three operating intervals on MJD 59404, 59411 and 59416, as shown in Fig. 1. The resulting uncertainties are represented in the following table according to Circular T notation:

Period of Estimation (MJD)	$\overline{y(\text{UTC}(\text{NICT}))}$	$u_A$	$u_B$	$u_{A/\text{Lab}}$	$u_{B/\text{Lab}}$	$u_{\text{Srep}}$	uptime
59399 – 59424	-42.70	0.45	0.77	4.20	0.80	4	1.1%
Effect	Uncertainty	$u_A$	$u_B$				
$u_{A/\text{Sr}}$	0.45	✓					
$u_B$	0.77		✓				
HM: linear trend estimation	3.31			✓			
HM: stochastic noise	2.58			✓			
Optical-microwave comparison / microwave transfer	0.80				✓		
Uncertainty of Sr as SRS	4					✓	

Table 1. Results of evaluation. All number are in parts of  $10^{-16}$ .

The evaluation employs the recommended value of the <sup>87</sup>Sr clock transition as a secondary representation of the definition of the second:  $\nu(^{87}\text{Sr}) = 429\,228\,004\,229\,873.0$  Hz with its relative standard uncertainty of  $u_{\text{Srep}} = 4 \times 10^{-16}$ , determined by the 21<sup>st</sup> CCTF in June 2017.

$u_A$  is the Type A uncertainty of NICT-Sr1 as an optical standard. It represents the statistical uncertainty determined by interleaved measurements [1].

$u_B$  is the Type B uncertainty of NICT-Sr1 [1 – 3], including the uncertainty of the gravitational redshift.

$u_{A/\text{Lab}}$  and  $u_{B/\text{Lab}}$  represent the uncertainty due to the link between NICT-Sr1 and the HM [2, 3]. They consist of

Type B uncertainty  $u_{B/\text{Lab}} = 7.95 \times 10^{-17}$  due to the frequency comparison between microwave and optical signals, including distribution of the microwave signals, and

Type A uncertainty  $u_{A/\text{Lab}} = 4.20 \times 10^{-16}$  representing the linear trend estimation of the HM ( $u_{l/\text{trend}}$ ) in addition to the uncertainty due to the stochastic noise of the HM during unobserved intervals ( $u_{l/\text{stoch}}$ ).

## 1. Evaluation of the frequency of hydrogen maser $\text{HM}_{1402012}$ with respect to NICT-Sr1 over 25 days

The details of NICT-Sr1 are described in [1, 2]. The Sr atoms were laser-cooled using a two-stage laser cooling technique and loaded to a vertically oriented one-dimensional optical lattice.

We transfer the HM behavior to a recently installed Er: fiber comb by stabilizing a heterodyne beat between the 37<sup>th</sup> harmonic of the frequency comb's approximately 250 MHz repetition rate and a 9.3 GHz signal from a dielectric resonator oscillator phase-locked to the 100 MHz signal of  $\text{HM}_{1402012}$ . The optical frequency reference at 698 nm supplied by NICT-Sr1 is measured as a beat with a frequency-doubled output branch of the comb. The comb repetition rate is detected at the output of the same output branch to reduce variations in relative phase, and the system incorporates a temperature-stabilized baseplate to maintain stable optical path lengths. The phase-locked beat signals for carrier-envelope offset and repetition rate are monitored by zero-deadtime counters.

The frequency comb system used in previous measurements operates simultaneously to confirm the measurement results. In this system, the repetition rate is stabilized by phase-lock of the heterodyne beat between the 82<sup>nd</sup> harmonic of the frequency comb's approximately 100 MHz repetition rate and an 8.2 GHz dielectric resonator oscillator phase-locked to the 100 MHz signal of  $\text{HM}_{1402012}$ . A transfer laser at 1397 nm is frequency-doubled by a PPLN waveguide. Its output is separated into a visible component used to phase-lock the laser to the optical frequency reference at 698 nm supplied by NICT-Sr1, and a residual infrared component that generates a beat signal with the frequency comb. This beat is counted both directly and through a tracking oscillator to allow rejection of cycle slips.

Both comb systems show agreement at a level below the reported statistical uncertainties.

The fractional deviation of the HM frequency from its nominal value is stored pre-averaged in 10 s bins. To assign appropriate weights, a statistical uncertainty determined from a daily stability estimate and the number of contributing data points is also stored for each bin.

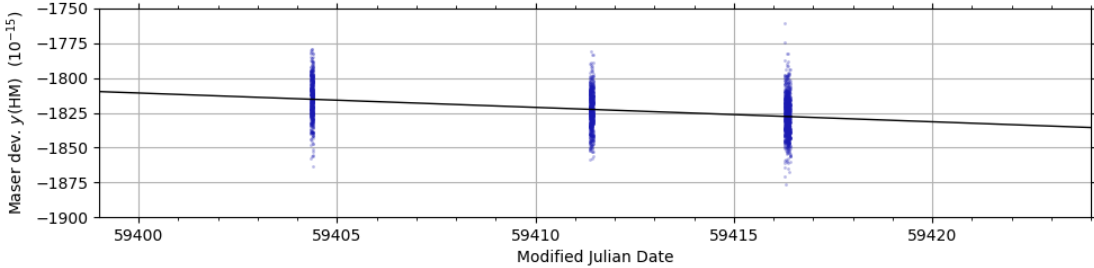


Fig. 1. Distribution of maser frequency measurements in terms of fractional deviation  $y(\text{HM})$  from the nominal frequency. The black solid line indicates the linear fit used to obtain the mean value over the evaluation interval.

## 2. Treatment of stochastic noise during unobserved intervals

For intermittent clock operation, phase and frequency excursions of the HM during unobserved intervals contribute significant measurement uncertainty [2,4].

We confirm the stability of  $\text{HM}_{1402012}$  over the entire evaluation period by comparing its frequency to an ensemble average of  $\text{HM}_{1402003}$ ,  $\text{HM}_{1402004}$ ,  $\text{HM}_{1402012}$  and  $\text{HM}_{1402013}$ . Their relative phase is continuously monitored by the DMTD system. Because of some recent technical problems with

this system, which were corrected only just before this evaluation period, we chose not to apply an ensemble correction at this time.

We characterize the typical instability of a single maser based on evaluation of several years of continuous data using three-corner-hat methods, and find that for large averaging times it is well-described by an Hadamard variance  $\sigma_H^2(\tau) = a_{-1} + a_{-3} \tau^2$ . Here,  $a_{-1} = (2.1 \times 10^{-16})^2$  represents flicker frequency noise (FFN) [5] while the slow-varying noise that dominates the long-term instability through  $a_{-3} = (1.9 \times 10^{-22}/\text{s})^2$  is typically referred to as flicker-walk frequency modulation (FWFM) [6]. We follow the approach described in the supplement of ref. [7] to determine the uncertainty of extrapolating from the inhomogeneously distributed data to the full evaluation period. This yields a distribution-specific sensitivity to the HM's noise power spectral density (PSD) [7], which we obtain from the observed maser instabilities  $a_{-1}$  and  $a_{-3}$  through the relations

$$\text{FFN: } \sigma_H^2(\tau) = \frac{1}{2} \ln\left(\frac{256}{27}\right) h_{-1} \quad \text{for } S_y^{\text{FFN}} = h_{-1} f^{-1} \quad \text{and} \quad (1)$$

$$\text{FWFM: } \sigma_H^2(\tau) = \frac{16}{6} \pi \ln\left(\frac{3}{4} \cdot 3^{11/16}\right) h_{-3} \tau^2 \quad \text{for } S_y^{\text{FWFM}} = h_{-3} f^{-3} \quad , \quad (2)$$

according to refs. [6, 8]. Additional information on the procedure is available in ref. [4]. Despite the complexity of FFN and FWFM in the temporal domain, the noise can always be expressed as a sum over normally distributed sources, and there is no correlation between the noise encountered in separate masers. The maser ensemble therefore shows the same uncertainty reduction with  $N_{\text{ens}}^{-1/2}$  as other noise types.

For the present (inhomogeneous) measurement distribution and PSDs, this leads to the uncertainty contributions  $u_{1/\text{FFN}} = 2.48 \times 10^{-16}$  and  $u_{1/\text{FWFM}} = 7.3 \times 10^{-17}$ , which we include as  $u_{1/\text{stoch}} = 2.58 \times 10^{-16}$ .

### 3. Determination of statistical and systematic contributions to $u_{1/\text{Lab}}$

We determine a statistical uncertainty of the maser frequency measurement  $u_{\text{stat}} = 3.19 \times 10^{-16}$  from the residuals of a linear fit by extrapolating the Allan deviation from the region limited by white frequency noise (30–10 000 s) to the full length of available data.

When plotting the instability of the frequency measurements for HM<sub>1402012</sub> in terms of the Hadamard deviation, we expect an observed flicker floor  $\sigma_L^2 \approx a_{-1}$  due to FFN.

Since the barycenter of the data (approximately MJD 59413.05) differs from the midpoint (MJD 59411.5) of the evaluation period by 133 980 s, the uncertainty of the maser drift rate contributes an uncertainty of  $u_{\text{drift}} = 8.8 \times 10^{-17}$ , such that  $u_{1/\text{trend}} = (u_{\text{stat}}^2 + u_{\text{drift}}^2)^{1/2} = 3.31 \times 10^{-16}$ .

While intermittent measurements of the maser frequency are easily affected by phase shifts resulting from thermalization effects at the start of frequency comb operation, as well as from diurnal temperature variation, an investigation of data obtained over a ten-day interval [9] sets a limit of  $u_{B,1/\text{Lab}} = 7.95 \times 10^{-17}$  for the window of  $T_{\text{typ}} = 21\,600$  s (6 h) duration during which intermittent measurements are typically performed. Frequency combs and measurement instruments are operated continuously to avoid start-up effects.

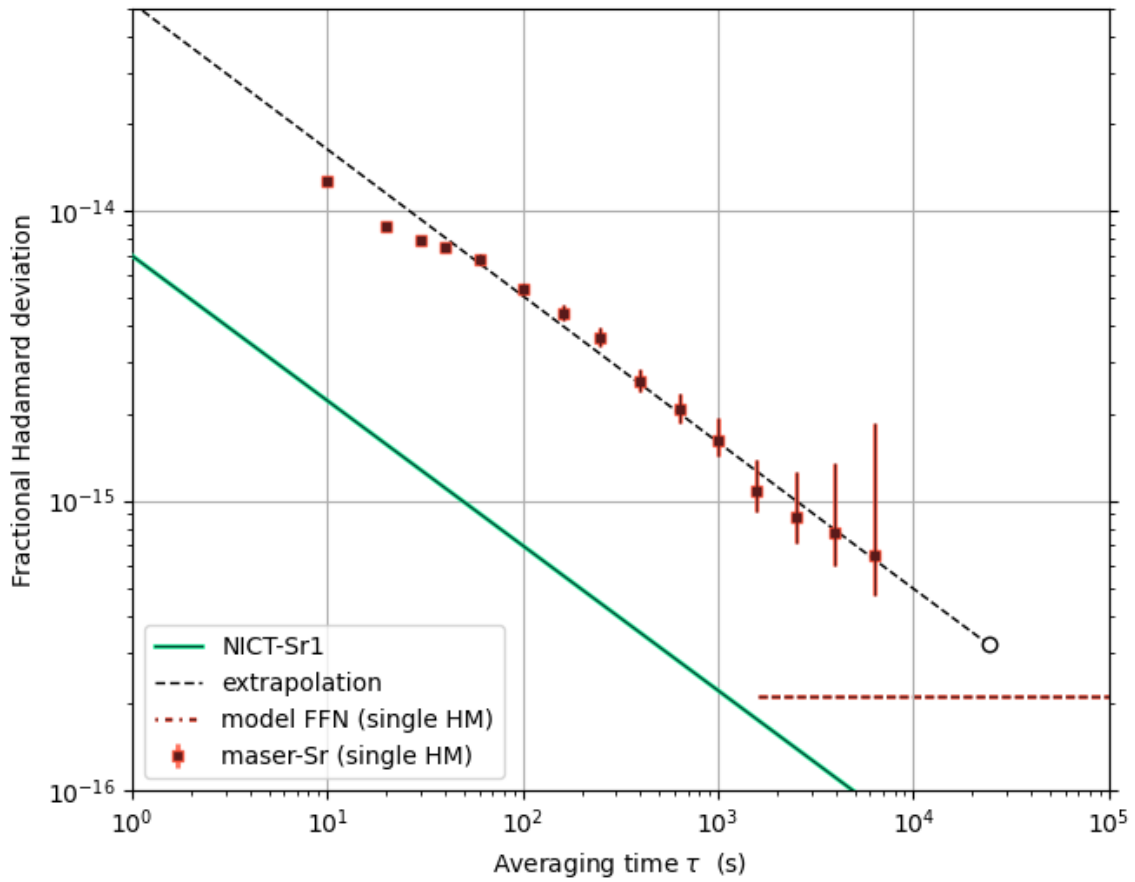


Fig.2. Instability of maser frequency measurements with respect to Sr. Red squares show the overlapping Hadamard deviation for the residuals from a linear fit. Gaps have been removed by contracting the data into a continuous interval. Error bars indicate  $1\sigma$  uncertainties, calculated for white frequency noise. The dashed line indicates the extrapolation to the full length of available data, used to obtain the statistical uncertainty indicated by the open mark. For measurement times longer than reported here, the Hadamard deviation falls to a level consistent with the maser stability model, indicated by the red dot-dashed line. The green line shows the instability contribution from NICT-Sr1.

#### 4. Frequency deviation of UTC(NICT)

The frequency difference between  $\text{HM}_{1402012}$  and UTC(NICT) over the evaluation period is typically calculated as

$$y(\text{UTC}(\text{NICT}) - \text{HM}_{1402012}) = (\delta_a - \delta_b)/T, \quad (3)$$

where  $\delta_a$  and  $\delta_b$  represent the time difference  $\text{UTC}(\text{NICT}) - \text{HM}_{1402012}$  at the beginning and end of an evaluation interval of length  $T$ . These values are continuously measured by the DMTD system [10] and reported to BIPM, where they are used in the EAL generation and made available at <https://webtai.bipm.org/ftp/pub/tai/data/>.

We recently observed some sporadic measurement errors of the DMTD system, and we investigate this by calculating frequency values for the intervals between hourly timing measurements. An independent measurement by a time-interval counter is used to confirm that any outliers are only present in the DMTD measurement, rather than the signals of either UTC(NICT) or  $\text{HM}_{1202012}$ .

Such outliers are then removed, and a linear fit is used to determine the mean value

$$y(\text{UTC}(\text{NICT}) - \text{HM}_{1402012}) = -1.818\,376 \times 10^{-12},$$

which we use to calculate  $\overline{y(\text{UTC}(\text{NICT}))}$  from  $\overline{y(\text{HM}_{1402012})}$ .

#### 5. Accuracy of NICT-Sr1

The systematic corrections and their uncertainties for NICT-Sr1 [1 – 3] are summarized below:

Effect	Correction ( $10^{-17}$ )	Uncertainty ( $10^{-17}$ )
Blackbody radiation	514.9	3.9
Lattice scalar / tensor	0	5.3
Lattice hyperpolarizability	-0.2	0.1
Lattice E2/M1	0	0.5
Probe light	0.1	0.1
Dc Stark	0.1	0.2
Quadratic Zeeman	50.8	0.3
Density	1.4	1.0
Background gas collisions	0	1.8
Line pulling	0	0.1
Servo error	0.9	2.5
Total	568.1	7.4
Gravitational redshift	-834.1	2.2
Total (with gravitational effect)	-266.0	7.7

Table 2. Systematic corrections and their uncertainties for NICT-Sr1.

## 6. References

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