Evaluation of $y(\mathrm{HM}_{1402015})$ with respect to NICT-Sr1 for the period MJD 58479 to 58509

The frequency of hydrogen maser $\mathrm{HM}_{1402015}$ was evaluated for the period from MJD 58479 to 58509 (December 27 – January 26) using secondary frequency standard NICT-Sr1. In terms of the fractional deviation from the nominal frequency, we find $\overline{y(\mathrm{HM}_{1402015})} = -8.07816 \times 10^{-13}$. The optical lattice clock acquired data for 34,998 s (1.4% of the total evaluation period) during three operating intervals on MJD 58479, 58492 and 58505. The resulting uncertainties are represented in the following table according to Circular T notation:

Period of Estimation (MJD)	$\overline{y(\mathrm{HM}_{1402015})}$	uA	uB	ul/Lab	uSrep
58479 – 58509	– 8078.16	0.37	0.77	3.18	4

Effect	Uncertainty	uA	uB	uA	uB	
uA/Sr	0.37	✓				
uB	0.77		✓			
HM: linear trend estimation	2.21			✓		
HM: stochastic phase noise	2.14			✓		
Optical-microwave comparison / microwave transfer	0.80				✓	
Uncertainty of Sr as SRS	4					✓

Table 1. Results of evaluation. All number are in parts of 10^{-16}.

The evaluation employs the recommended value of the ^{87}Sr clock transition as a secondary representation of the definition of the second: $\nu(^{87}\mathrm{Sr})$ = 429 228 004 229 873.0 Hz with its relative standard uncertainty of uSrep = 4×10^{-16}, determined by the 21st CCTF in June 2017.

uA is the Type A uncertainty of NICT-Sr1 as an optical standard. It represents the statistical uncertainty determined by interleaved measurements [1].

uB is the Type B uncertainty of NICT-Sr1 [1 – 3], including the uncertainty of the gravitational redshift.

ul/Lab = 3.18×10^{-16} is the uncertainty due to the link between NICT-Sr1 and the HM [2, 3]. It consists of the Type B uncertainty uB_l/Lab = 8.0×10^{-17} due to the frequency comparison between microwave and optical signals, including distribution of the microwave signals, and the Type A uncertainty uA_l/Lab = 3.08×10^{-16} representing the linear trend estimation of the HM (ul/HMtrend) and the additional uncertainty due to the stochastic noise of the HM during unobserved intervals (ul/HMstoch).

1. Evaluation of the frequency of hydrogen maser $\mathrm{HM}_{1402015}$ with respect to NICT-Sr1 over 30 days

The ^{87}Sr optical lattice clock NICT-Sr1 was operated in the same mode during all operating intervals. The details of NICT-Sr1 are described in [1, 2]. The Sr atoms were laser-cooled using a two-stage laser cooling technique and loaded to a vertically oriented one-dimensional optical lattice. The optical frequency at the wavelength of 698 nm stabilized to NICT-Sr1 was down-converted to a microwave frequency using an Yb:fiber-based frequency comb. By stabilizing the comb to the optical reference with appropriately chosen frequency offsets, an optically generated microwave with a frequency of 9.25 GHz was derived from the thirty-seventh harmonic of the repetition rate (= 250 MHz), assuming an optical frequency of the clock transition according to the recommended value as a secondary representation of the definition of the second $\nu(^{87}\mathrm{Sr})$. The microwave frequency was then down-mixed to 50 MHz using a 9.2 GHz signal provided by an oscillator phase-locked to the 100 MHz signal of $\mathrm{HM}_{1402015}$. The down-mixed signal was counted and recorded every second by a zero-dead-time frequency counter referenced to the same HM.

The fractional deviation of the HM frequency from its nominal value is stored pre-averaged in 10 s bins. To assign appropriate weights, a statistical uncertainty determined from a daily stability estimate and the number of contributing datapoints is also stored for each bin.

2. Updated treatment of stochastic noise during unobserved intervals

For intermittent clock operation, phase and frequency excursions of the HM during unobserved intervals contribute significant measurement uncertainty [2]. For the current set of measurements, a smaller amount of data is available over the evaluation period. We therefore include a somewhat extended treatment of the stochastic noise.

To mitigate their overall effect, we now include a total of five HMs (instead of three) in the evaluation ($\mathrm{HM}_{1402004}$, $\mathrm{HM}_{1402012}$, $\mathrm{HM}_{1402013}$, $\mathrm{HM}_{1402014}$ and $\mathrm{HM}_{1402015}$). Their relative phase is continuously monitored by the dual mixer time difference (DMTD) system used in the generation of Japan Standard Time. We calculate the phase difference of each HM from the ensemble average phase and confirm the absence of abnormal behavior during the evaluation period. We then determine the frequency of $\mathrm{HM}_{1402015}$ with respect to the ensemble. By subtracting frequency offset and linear trend from this relative frequency, we obtain residuals that approximate the instantaneous deviation of $\mathrm{HM}_{1402015}$ from a pure linear drift, while summing to zero over the evaluation period. We use these residuals (calculated for a set of one-hour intervals) to correct the HM frequency measured with respect to the Sr clock. The result is an improved representation of the mean frequency and linear drift of $\mathrm{HM}_{1402015}$ over the complete evaluation period. A weighted linear fit is applied to the corrected data to find the frequency corresponding to the midpoint of the 30-day interval.

We characterize the typical instability of a single maser based on evaluation of several years of continuous data using three-corner-hat methods, and find that it is well-described by an Allan variance $\sigma_y^2(\tau) = a_0\,\tau^{-1} + a_{-1} + a_{-3}\,\tau^2$. Here, $a_0 = (7.0 \times 10^{-14})^2$ s corresponds to white frequency noise and $a_{-1} = (2.1 \times 10^{-16})^2$ to flicker frequency noise (FFN) [4]. The slow-varying noise that dominates the long-term instability through $a_{-3} = (6.3 \times 10^{-22}/\mathrm{s})^2$ is commonly referred to as flicker walk frequency modulation (FWFM) [5].

The latter dominates the uncertainty for longer interruptions of the measurement. For each gap of duration τ_g, we assign a variance of $\sigma^2 = a_{-3}\,\tau_g^2 / N_e$, where the division by $N_e = 5$ represents the effect of averaging over an ensemble of five masers. By weighting the individual variances according to their contribution to the evaluation period, we find an overall uncertainty ul/FWFM $= 1.9 \times 10^{-16}$.

The characteristic floor of the Allan deviation due to flicker frequency noise represents rather complex temporal correlations. To take these into account for an arbitrary distribution of dead-time, we approximate the noise contribution by the output of a Gardner/Voss 1/f-noise generator as originally described in [6]. This generator employs a range of simple, normal-distributed noise sources updating their outputs at different intervals, and yields a simple analytical representation of the output state at any given instant. This allows the difference between the true average over a designated period and the average over an arbitrary subset of samples to be evaluated in terms of the resulting variance without resorting to Monte-Carlo methods. For the distribution encountered here, the result is an uncertainty ul/FFN = 1.01 $(a_{-1}/N_e)^{1/2}$ = 9.7 × 10^{-17} due to FFN. Despite its complexity in the temporal domain, the flicker noise can always be expressed as a sum over normally distributed sources, and there is no correlation between the noise encountered in separate masers. The maser ensemble therefore shows the same uncertainty reduction with $N_{ens}{}^{-1/2}$ as other noise types.

Combining the two contributions, we find ul/HMstoch = 2.14 × 10^{-16}.

Applying the evaluation used in previous reports, which does not distinguish ul/FWFN and ul/FFN, and does not account for the stability gain from referencing multiple masers, we find an effectively identical maser frequency with a somewhat increased uncertainty ul'/HMstoch = 3.68 × 10^{-16}.

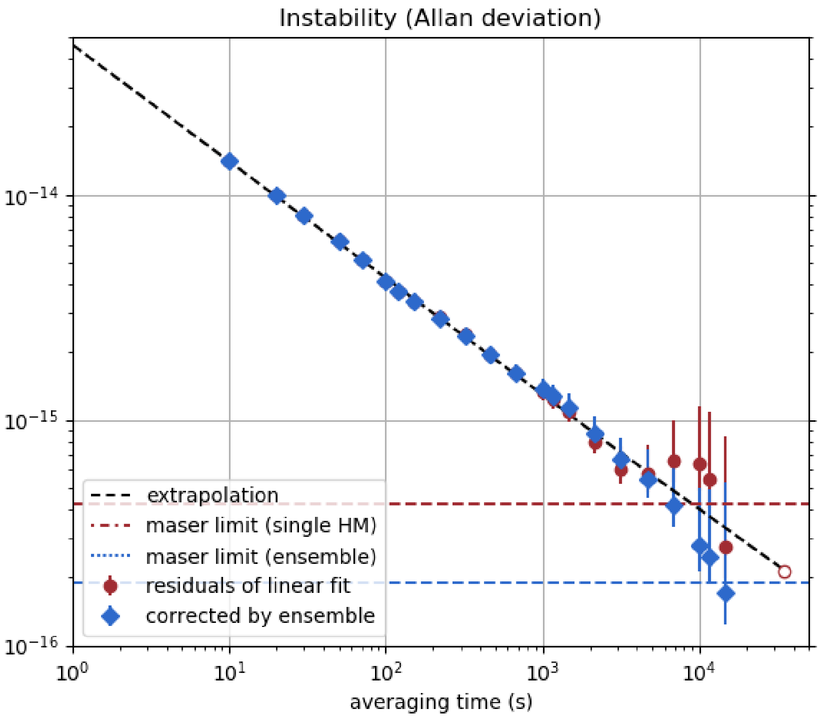

Fig.1. Instability of maser frequency measurements. Blue diamonds and red circles show the overlapping Allan deviation for the residuals from a linear fit with and without correction for deviation from linear drift of the HM, as determined using an ensemble of 5 HMs. Error bars indicate 1σ uncertainties assuming white frequency noise. The dashed line indicates the extrapolation to the full length of available data, used to obtain the statistical uncertainty. For long averaging times, the Allan deviation of the uncorrected data is consistent with the expected stability limit of $\sigma_1 \approx 4.3 \times 10^{-16}$ as shown by the red dot-dashed line. The ensemble stability reduces the expect stability limit for the corrected data by a factor of 1 / $(N_e)^{1/2}$, as indicated by the blue dotted line.

3. Determination of statistical and systematic contributions to ul/Lab

We determine a statistical uncertainty of the maser frequency measurement as $u_{\mathrm{stat}} = 2.15 \times 10^{-16}$ from the residuals of a linear fit by extrapolating the Allan deviation from the region limited by white frequency noise (30-3000 s) to the full length of available data (Fig. 1).

When evaluating the data for only $\mathrm{HM}_{1402015}$, we expect a stability limit according to $\sigma_{\mathrm{L}}^2 \approx a_{-1} + a_{-3}\,(T/2)^2$ for an evaluation period $T = 30 \times 86{,}400$ s. After applying corrections for nonlinear drift, we expect the results to represent the stability of the entire ensemble used to determine the corrections. As shown in Fig. 1, the observed stabilities are indeed consistent with $\sigma_{\mathrm{L}} \approx 4.3 \times 10^{-16}$ before correction, and $\sigma_{\mathrm{L}} / N_{\mathrm{e}}^{1/2} \approx 1.9 \times 10^{-16}$ afterwards.

Since the midpoint (MJD 58494) of the evaluation period differs from the barycenter of the data by 193,544 s, the uncertainty of the maser drift rate contributes an uncertainty of $u_{\mathrm{drift}} = 5.0 \times 10^{-17}$, such that ul/HMtrend = $(u_{\mathrm{stat}}^2 + u_{\mathrm{drift}}^2)^{1/2} = 2.21 \times 10^{-16}$.

While intermittent measurements of the maser frequency are easily affected by phase shifts resulting from thermalization effects at the start of frequency comb operation, as well as from diurnal temperature variation, an investigation of data obtained in a near-continuous measurement over a ten-day interval [7] sets an improved limit of uB_l/Lab = 7.95×10^{-17} for a resulting systematic frequency shift.

4. Accuracy of NICT-Sr1

The systematic corrections and their uncertainties for NICT-Sr1 [1 – 3] are summarized below:

Effect	Correction (10^{-17})	Uncertainty (10^{-17})
Blackbody radiation	508.5	3.7
Lattice scalar / tensor	0	5.3
Lattice hyperpolarizability	–0.2	0.1
Lattice E2/M1	0	0.5
Probe light	0.1	0.1
Dc Stark	0.1	0.2
Quadratic Zeeman	51.3	0.3
Density	1.3	2.7
Background gas collisions	0	1.8
Line pulling	0	0.1
Servo error	5.3	1.5
Total	566.3	7.4
Gravitational redshift	–834.1	2.2
Total (with gravitational effect)	–267.8	7.7

Table 2. Systematic corrections and their uncertainties for NICT-Sr1.

5. References

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