Evaluation of $y(\mathrm{HM}_{1402015})$ with respect to NICT-Sr1 for the period MJD 58454 to 58464

During a measurement campaign between MJD 58454 and 58464 (2nd December – 12th December 2018), the frequency of hydrogen maser $\mathrm{HM}_{1402015}$ was evaluated using secondary frequency standard NICT-Sr1. In terms of the fractional deviation from the nominal frequency, we find $\overline{y(\mathrm{HM}_{1402015})} = -7.7968 \times 10^{-13}$ over the estimation period. The optical lattice clock acquired data for 779,853 s (90.3% of the total evaluation period). The resulting uncertainties are represented in the following table according to Circular T notation:

Period of Estimation (MJD)	$\overline{y(\mathrm{HM}_{1402015})}$	uA	uB	ul/Lab	uSrep
58454 – 58464	–7796.8	0.08	0.79	0.47	4

Effect	Uncertainty	uA	uB	uA	uB	
uA/Sr	0.08	✓				
uB	0.79		✓			
HM: linear trend estimation	0.39			✓		
HM: stochastic phase noise	0.26			✓		
Optical-microwave comparison / microwave transfer	0.01				✓	
Uncertainty of Sr as SRS	4					✓

Table 1. Results of evaluation. All number are in parts of 10^{-16}.

The evaluation employs the recommended value of the ^{87}Sr clock transition as a secondary representation of the definition of the second: $\nu(^{87}\mathrm{Sr}) = 429\,228\,004\,229\,873.0$ Hz with its relative standard uncertainty of uSrep = 4×10^{-16}, determined by the 21st CCTF in June 2017.

uA is the Type A uncertainty of NICT-Sr1 as an optical standard. It represents the statistical uncertainty determined by interleaved measurements [1].

uB is the Type B uncertainty of NICT-Sr1 [1 – 3], including the uncertainty of the gravitational redshift.

ul/Lab is the uncertainty due to the link between NICT-Sr1 and the HM [2, 3]. It consists of the Type A uncertainty uA_l/Lab = 4.7×10^{-17} representing the linear trend estimation of the HM (ul/HMtrend) combined with the additional uncertainty due to the stochastic noise of the HM during unobserved periods (ul/HMstoch), and the Type B uncertainty uB_l/Lab = 1.2×10^{-18} due to the frequency comparison between microwave and optical signals, including distribution of the microwave signals.

1. Evaluation of the frequency of hydrogen maser $\mathrm{HM}_{1402015}$ with respect to NICT-Sr1 over 10 days

The ^{87}Sr optical lattice clock, NICT-Sr1, was operated in the same mode during the entire operating interval. The details of NICT-Sr1 are described in [1, 2]. The Sr atoms were laser-cooled using a two-stage laser cooling technique and loaded to a vertically oriented one-dimensional optical lattice. The optical frequency at the wavelength of 698 nm stabilized to NICT-Sr1 was down-converted to a microwave frequency using an Yb:fiber-based frequency comb. By stabilizing the comb to the optical reference with appropriately chosen frequency offsets, an optically generated microwave with a frequency of 9.25 GHz was derived from the thirty-seventh harmonic of the repetition rate (= 250 MHz), assuming an optical frequency of the clock transition according to the recommended value as a secondary representation of the definition of the second $\nu(^{87}Sr)$. The microwave frequency was then down-mixed to 50 MHz using a 9.2 GHz signal provided by an oscillator phase-locked to the 100 MHz signal of $HM_{1402015}$. The down-mixed signal was counted and recorded every second by a zero-dead-time frequency counter referenced to the same HM. After finding the fractional deviation of the HM frequency from its nominal value, the frequency corresponding to the midpoint of the ten-day interval was determined from a weighted linear fit to data pre-averaged in 10 s bins. In determining the appropriate weights, we consider a daily stability estimate based on the Allan deviation of the HM frequency measurement.

2. Determination of statistical and systematic contributions to ul/Lab

The characterization of the maser frequency is limited by white phase and frequency noise that is typically not sampled identically by different measurements. We determine this contribution from the residuals of the linear fit. For averaging times $\tau > 30{,}000$ s, the Allan deviation of the residuals (Fig. 1) converges on the maser's flicker noise floor. We therefore determine the statistical uncertainty as $u_{stat} = 3.9 \times 10^{-17}$ by extrapolating the Allan deviation as $\propto \tau^{-1/2}$ from $\tau = 10{,}000$ s to the full length of available data. The reference value of $\sigma_F = 2.2 \times 10^{-16}$ indicated by the blue dot-dashed line in the figure represents the maser flicker floor as it is expected to appear in the Allan deviation of the residuals of the linear fit. This value was obtained using a four-corner hat method applied to the Hadamard deviations of the frequency differences for a subset of NICT's masers that was selected for good long-term stability.

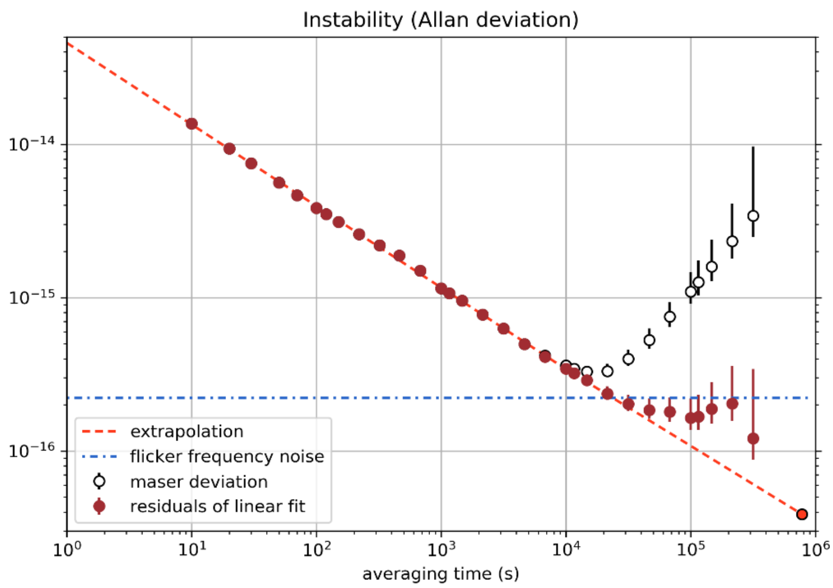

Fig.1. Instability of maser frequency measurements. Black open points represent the measured maser frequency without drift correction, while red solid points show the residuals from a linear fit. Values are overlapping Allan deviations, with error bars indicating 1σ uncertainties assuming white frequency noise. For long averaging times, the Allan deviation is consistent with a maser flicker frequency noise of $\sigma_F = 2.2 \times 10^{-16}$ as shown by the blue dot-dashed line. The overall statistical uncertainty is obtained by extrapolation to the full length of available data as indicated by the orange dashed line.

Since the midpoint (MJD 58459) of the measurement period differs from the barycenter of the data by only 3,824 s, the uncertainty of the maser drift rate contributes a negligible uncertainty of $u_{\mathrm{drift}} = 7.5 \times 10^{-19}$, such that ul/HMtrend = $(u_{\mathrm{stat}}^2 + u_{\mathrm{drift}}^2)^{1/2} = 3.9 \times 10^{-17}$. We estimate ul/HMstoch by considering the flicker-noise induced phase uncertainty $\tau\ \sigma_{\mathrm{F}}/(\ln 2)^{1/2}$ over an unobserved interval of length τ [4]. We conservatively combine all unobserved intervals in the overall measurement of length T = 864,000 s into a single interval of τ = 84,147 s, from which we estimate ul/HMstoch = $\tau\ \sigma_{\mathrm{F}}$ / $(\ln 2)^{1/2}$ / T = 2.6×10^{-17}. Although the actual uncertainty is expected to be lower due to the partial cancellation of uncorrelated noise processes, the precise value is uncritical to this evaluation as ul/HMstoch is dominated by ul/HMtrend and the satellite link uncertainty over 10 days of 7.0×10^{-16}.

While intermittent measurements of the maser frequency are easily affected by phase shifts resulting from diurnal temperature variation and thermalization effects at the start of the frequency comb operation, these effects are strongly suppressed in a continuous measurement covering the full 24 h cycle. Our typical uB_l/Lab = 1×10^{-16} for 10,000 s yields uB_l/Lab = 1.2×10^{-18} for the extended measurement interval of 864,000 s.

3. Accuracy of NICT-Sr1

The systematic corrections and their uncertainties for NICT-Sr1 [1 – 3] are summarized below:

Effect	Correction (10^{-17})	Uncertainty (10^{-17})
Blackbody radiation	509.0	2.6
Lattice scalar / tensor	0	5.3
Lattice hyperpolarizability	–0.2	0.1
Lattice E2/M1	0	0.5
Probe light	0.1	0.1
Dc Stark	0.1	0.2
Quadratic Zeeman	51.3	0.3
Density	2.0	4.1
Background gas collisions	0	1.8
Line pulling	0	0.1
Servo error	0.4	1.5
Total	562.7	7.6
Gravitational redshift	–834.1	2.2
Total (with gravitational effect)	–271.4	7.9

Table 2. Systematic corrections and their uncertainties for NICT-Sr1.

4. References

[1] H. Hachisu and T. Ido, “Intermittent optical frequency measurements to reduce the dead time uncertainty of frequency link,” Jpn. J. Appl. Phys. **54**, 112401 (2015).

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