Evaluation of $y(\mathrm{HM}_{1402003})$ with respect to NICT-Sr1 for the period MJD 58149 to 58174

During a measurement campaign between MJD 58149 and 58174 (31st January – 25th February 2018), the frequency of hydrogen maser $\mathrm{HM}_{1402003}$ was evaluated using secondary frequency standard NICT-Sr1. In terms of the fractional deviation from the nominal frequency, we find $\overline{y(\mathrm{HM}_{1402003})} = 5.0913 \times 10^{-13}$ over the estimation period. The optical lattice clock was operated for 57,745 s (2.7% of the total evaluation period). The mean fractional deviation of the maser $\overline{y(\mathrm{HM}_{1402003})}$ over the evaluation period was derived from that of average of the three masers ($\mathrm{HM}_{1402003}$, $\mathrm{HM}_{1402012}$, and $\mathrm{HM}_{1402015}$) over the period which was estimated from a linear fit of measurements during 5 operating intervals. The resulting uncertainties are represented in the following table according to Circular T notation:

Period of Estimation (MJD)	$\overline{y(\mathrm{HM}_{1402003})}$	uA	uB	ul/Lab	uSrep
58149 – 58174	5091.3	0.29	0.73	2.93	4

Effect	Uncertainty	uA	uB	uA	uB	
uA/Sr	0.29	✓				
uB	0.73		✓			
HM: linear trend estimation	1.8			✓		
HM: stochastic phase noise	2.03			✓		
DMTD system	0.45			✓		
Optical-microwave comparison / microwave transfer	1				✓	
Uncertainty of Sr as SRS	4					✓

Table 1. Results of evaluation. All number are in parts of 10^{-16}.

The evaluation employs the recommended value of the ^{87}Sr clock transition as a secondary representation of the definition of the second: $\nu(^{87}\mathrm{Sr})$ = 429 228 004 229 873.0 Hz with its relative standard uncertainty of uSrep = 4×10^{-16}, determined by the 21st CCTF in June 2017.

uA is the Type A uncertainty. It represents the statistical uncertainty in NICT-Sr1, determined by interleaved measurements [1].

uB is the Type B uncertainty of NICT-Sr1 [1 – 3], including the uncertainty of the gravitational redshift.

ul/Lab is the uncertainty due to the link between NICT-Sr1 and HMs [2, 3]. It consists of the Type A uncertainty uA_l/Lab = 2.75×10^{-16} representing the linear trend estimation of the HMs (ul/HMtrend), the stochastic phase noise of the HM (ul/HMstoch), as well as the measurement uncertainty for the frequency difference between HMs (ul/DMTD), and the Type B uncertainty uB_l/Lab = 1×10^{-16} due to the frequency comparison between microwave and optical signals, including distribution of the microwave signals:

- ul/HMtrend is conservatively determined to be 1.8×10^{-16} from the uncertainty of the analytical estimation $\sigma_p/(N + 1)^{1/2}$ described in Ref [3] since the linear fitting error to the five data blocks separated by approximately one week is 1.2×10^{-16} in this campaign.
- ul/HMstoch is estimated using $\tau \sigma_F/(\ln 2)^{1/2}$, which is the induced phase uncertainty over a non-operation time τ for a flicker frequency noise σ_F [4]. Thus, a non-operation time of 7 days results in a phase uncertainty of 0.23 ns with $\sigma_F = 3.2 \times 10^{-16}$ estimated using the Hadamard variance. Over the 25-day interval, the uncertainty of the stochastic part ul/HMstoch in the one-month mean frequency is then estimated as $0.23 \times 10^{-9} \times (25/7)^{1/2}/(86400 \times 25) = 2.0 \times 10^{-16}$.
- ul/DMTD for five NICT-Sr1 operations is estimated as $1 \times 10^{-16}/5^{1/2} = 4.5 \times 10^{-17}$ as the noise of the DMTD system is 1×10^{-16} over an averaging time of 10^4 s.

1. Evaluation of the frequency of hydrogen maser $HM_{1402003}$ with respect to NICT-Sr1 over a month

The ^{87}Sr optical lattice clock, NICT-Sr1, was operated in the same mode during all operating intervals. The details of NICT-Sr1 are described in [1, 2]. The Sr atoms were laser-cooled using a two-stage laser cooling technique and loaded to a vertically oriented one-dimensional optical lattice. The optical frequency at the wavelength of 698 nm stabilized to NICT-Sr1 was down-converted to a microwave frequency using an Yb:fiber-based frequency comb. By stabilizing the comb to the optical reference with appropriately chosen frequency offsets, an optically generated microwave with a frequency of precisely 9.25 GHz was derived from the thirty-seventh harmonic of the repetition rate (= 250 MHz), assuming an optical frequency of the clock transition according to the recommended value as a secondary representation of the definition of the second $\nu(^{87}\mathrm{Sr})$. The microwave frequency was then down-mixed to 50 MHz using a 9.2 GHz signal provided by an oscillator phase-locked to the 100 MHz signal of $HM_{1402003}$. The down-mixed signal was counted and recorded every second by a zero-dead-time frequency counter referenced to the same HM. After finding the fractional deviation of the HM frequency from its nominal value, the mean for the target month was determined from five data blocks homogeneously distributed over the period, each consisting of 10 data points pre-averaged over 1000 s. To mitigate the effect of sporadic phase excursions of a specific HM [2], two additional HMs were included in the analysis through their frequency differences from the reference maser, as continuously monitored by the dual mixer time difference (DMTD) system used in the generation of Japan Standard Time. The resulting ensemble averages were evaluated by linear fitting to find the one-month mean frequency. The reported value of $\overline{y(\mathrm{HM}_{1402003})}$ with respect to NICT-Sr1 was then derived with negligible additional uncertainty using DMTD data for the full period.

2. Accuracy of NICT-Sr1

The systematic corrections and their uncertainties for NICT-Sr1 [1 – 3] are summarized below:

Effect	Correction (10^{-17})	Uncertainty (10^{-17})
Blackbody radiation	508.7	2.7
Lattice scalar / tensor	0	5.3
Lattice hyperpolarizability	–0.2	0.1
Lattice E2/M1	0	0.5
Probe light	0.1	0.1
Dc Stark	0.1	0.2
Quadratic Zeeman	51.2	0.3
Density	3.5	2.8
Background gas collisions	0	1.8
Line pulling	0	0.1
Servo error	0.03	1.5
Total	563.4	7.0
Gravitational redshift	–834.1	2.2
Total (with gravitational effect)	–270.7	7.3

Table 2. Systematic corrections and their uncertainties for NICT-Sr1.

3. References

[1] H. Hachisu and T. Ido, "Intermittent optical frequency measurements to reduce the dead time uncertainty of frequency link," Jpn. J. Appl. Phys. **54**, 112401 (2015).

[2] H. Hachisu, G. Petit, F. Nakagawa, Y. Hanado and T. Ido, "SI-traceable measurement of an optical frequency at low 10^{-16} level without a local primary standard," Opt. Express **25**, 8511 (2017).

[3] H. Hachisu, F. Nakagawa, Y. Hanado and T. Ido, "Months-long real-time generation of a time scale based on an optical clock," Sci. Reports **8**, 4243 (2018).

[4] D. Allan, "Time and frequency (time-domain) characterization, estimation, and prediction of precision clock and oscillators," IEEE UFFC **34**, 647 (1987).