

IEN-CsF1 TAI EVALUATION

MJD 53559-53584 (8 July-2 August 2005)

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Introduction

During the period MJD 53559.0-53584.0, IEN has evaluated the frequency of its Hydrogen Maser IEN-HM2 (BIPM code 1401102) using the Cs fountain IEN-CsF1. The evaluation procedure of the fountain standard follows the general procedures reported in [1]; we report here details on the Type A and Type B uncertainty evaluation, together with the internal transfer uncertainty (including the contribution of dead time).

Even if the fountain set-up and measurement procedure is not substantially changed with respect to the last measurements (periods MJD 53304-53324 and MJD 53404-53414), we provide here a new evaluation of the microwave leakage contribution to the accuracy, the specification of the uncertainty due to the spin exchange evaluation in Type A and Type B contributions and a more accurate evaluation of the uncertainty introduced by the fountain dead time.

IEN-CsF1 Accuracy Evaluation

Black Body Radiation Shift $\Delta\nu_{\text{BBR}}$

The evaluation of the Blackbody Radiation (BBR) Shift $\Delta\nu_{\text{BBR}}$ requires the effective BBR temperature T experienced by the atoms along their ballistic flight. For the calculation of T , we interpolate the temperature data coming from four thermocouples positioned along the drift tube with a polygonal curve and then we calculate the average radiation temperature experimented by the atoms at a given position (integrated over the solid angle); in this way it is possible to take into account also the effect of the two “holes” in the blackbody radiator, the upper window and the hole in the microwave cavity. The values obtained at different elevations inside the fountain drift tube are then used to calculate the time averaged radiation temperature seen by the atoms along their ballistic flight. See the discussion reported in [2] for details.

To evaluate $\Delta\nu_{\text{BBR}}$ from the effective temperature T we follow the well known relation discussed for example in [2] and reported here below; the leading coefficient β here used is calculated using results presented in [3]; the coefficient ε is taken from [4].

$$\Delta\nu_{\text{BBR}} = \beta (T/300)^4 \cdot [1 + \varepsilon(T/300)^2]$$

$$\beta = (-1.711 \pm 0.003) \cdot 10^{-14}$$

$$\varepsilon = 0.014$$

$$T = 69.9 \pm 0.3 \text{ }^\circ\text{C} = 343.1 \pm 0.3 \text{ K}$$

$$\Delta\nu_{\text{BBR}} = (-29.8 \pm 0.1) \cdot 10^{-15}$$

Gravitational Red Shift Δv_{RS}

The absolute orthometric height (h) of the IEN-CsF1 location is calculated using a geodetic height and a Geoid model. The geodetic height with respect to the ellipsoid WGS84 is provided by the IEN GPS geodetic receiver. The Geoid height (EGM96) with respect to the WGS84 coordinate was calculated using the National Geospatial-Intelligence Agency (NGA) EGM96 Geoid calculator, available at the URL <http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm96/intpt.htm>. The height difference between the GPS antenna position and the fountain location was obtained as a result of a direct measurement. Reference for the proportional coefficient γ value is [5].

$$\Delta v_{RS} = \gamma \cdot h$$

$$\gamma = 1.09 \cdot 10^{-16} \text{ m}^{-1}$$

$$h = 242 \pm 1 \text{ m}$$

$$\Delta v_{RS} = (26.4 \pm 0.1) \cdot 10^{-15}$$

Quadratic Zeeman Shift Δv_Z

The effective C-field experienced by the atoms (B_0) along their trajectory is calculated (see [1] for details) from a field map which is obtained measuring the low frequency magnetic resonance transitions when the atoms are at the apogee; the map is completed launching the atoms at different apogee heights.

The C-field map obtained immediately before this evaluation period is reported in the figure 1 and it was used to calculate the quadratic Zeeman shift, following the relation below reported. Reference for the value of the quadratic Zeeman constant K is [5].

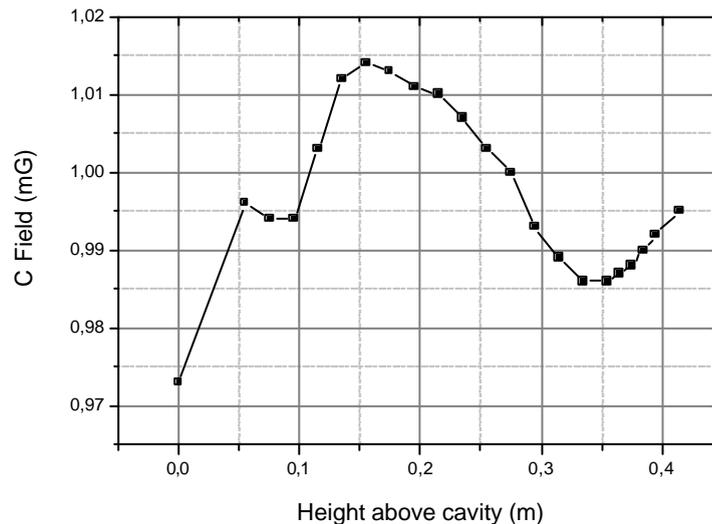


Figure 1. C-field map.

The heater used to frequency tune the Ramsey cavity and to stabilize the drift tube temperature is powered with an audio-frequency generator (100 kHz) to avoid the penetration of the generated magnetic field inside the drift tube. To prevent the occurrence of a dynamic end-to-end

phase shift [6], caused by a temperature modulation of the cavity synchronous with the Ramsey cycle, the heater is continuously powered during the whole operation cycle of the fountain. Although the magnetic field produced by the audio frequency generator is shielded by several skin depths, a quadratic Zeeman shift could arise by the RMS value of the residual magnetic field which penetrates inside the tube. Differential measurements (see [2] for details) provide an upper value to the shift due to the residual quadratic Zeeman effect of $0.4 \cdot 10^{-15}$. This is the leading contribution to the Zeeman shift uncertainty.

$$\Delta\nu_Z = K \cdot B_0^2$$

$$K = 427.45 \text{ Hz/T}^2$$

B_0 , C-field as calculated with the map

$$\Delta\nu_Z = (46.1 \pm 0.4) \cdot 10^{-15}$$

Collisional Shift

The collisional shift was evaluated with differential measurements (which also contributes to the fountain evaluation measurements), comparing the fountain frequency when it operates at high and low density conditions. As it was reported in [1], direct proportionality between density and total number of detected atoms is assumed. The differential measurements provide a collisional coefficient which is then used to correct the spin-exchange shift in each measurement run with respect to the average detected atom number of the run itself. As the differential measurement runs lasted for a relatively short interval during the fountain evaluation period, the collisional correction coefficient has a relative uncertainty of 35% and this uncertainty contribution is included in the Type A uncertainty budget.

A further contribution to collisional shift uncertainty was reported in the Type B budget. This contribution is mainly due to the possible variation of detection efficiency during the evaluation period and it is estimated as the 20% of the average density correction of the whole period.

Other Shifts

The actual influence of shifts resulting from several physical and technical effects was carefully investigated during the most recent history of IEN-CsF1. The contribution of these shifts is either negligible or not easily modelled and then they are not corrected for. For these effects only an uncertainty contribution is provided, reflecting the estimation of their maximum values during the fountain operative conditions.

These shifts, either theoretically estimated or measured, are [1]

- Resonant light shift
- Distributed cavity shift
- Dynamic end-to-end phase shift
- Cavity pulling and under-threshold maser retroaction
- Relativistic Doppler shift
- Synthesizer and numerical loop errors
- Microwave leakage and power-related shifts

Before the evaluation run here reported, many tests were conducted in order to estimate the shift and the uncertainty contributions of the microwave leakage during the operation of IEN-CsF1. All the possible sources of microwave leakage were carefully surveyed and then shielded, when possible. After that, some leverage tests, conducted operating the fountain with a high microwave power level, provided an estimation of the possible leakage shift.

As it was recently reported [7], the relation between the microwave operating power and the leakage induced shift is not linear and can be dramatically different if the leakage occurs between the two atom interrogations or just before the detection stage.

For these reasons, leverage tests were designed following the theory reported in [7], just to avoid ambiguous results, and different tests were conducted to estimate the shift due to the leakage during different stages of the fountain cycle.

The estimation of the microwave leakage shift is zero with an uncertainty of $0.6 \cdot 10^{-15}$

Summary of accuracy evaluation

Effect	Shift (10^{-15})	Uncertainty (10^{-15})
2 nd order Zeeman Shift	+46.1	0.4
Blackbody Radiation Shift	-29.8	0.1
Gravitational Red Shift	+26.4	0.1
Microwave Leakage Shift	--	0.6
Collisional Shift (Systematic)	--	0.5
Other shifts	--	0.2
Total	+42.7	0.9

Table 1. Summary of corrected and uncorrected shifts and uncertainty budget for IEN-CsF1, period MJD 53559-53584.

Evaluation of the average frequency $y(IEN-CsF1)-y(HM2)$

During the reported evaluation period, at IEN only one H-maser was running (BIPM code 1401102), as the other one (BIPM code 1401101) was unavailable due to maintenance.

The average frequency $y(IEN-CsF1)-y(HM2)$ over the period MJD 53559.0-53584.0 was calculated with a linear fit on the $y(IEN-CsF1)-y(HM2)$ data, coming from each individual fountain run and corrected for the collisional shift. As these data have different Type A uncertainties, we used a weighted least square algorithm. The fit method was chosen because fountain dead time is unavoidable during the evaluation period, and the dead time intervals are neither evenly spaced nor symmetric with respect to the center of the evaluation period. In these conditions, dead time would have biased an estimation derived by a standard average [8]. Epoch distribution of fountain dead time is reported in Figure 2.

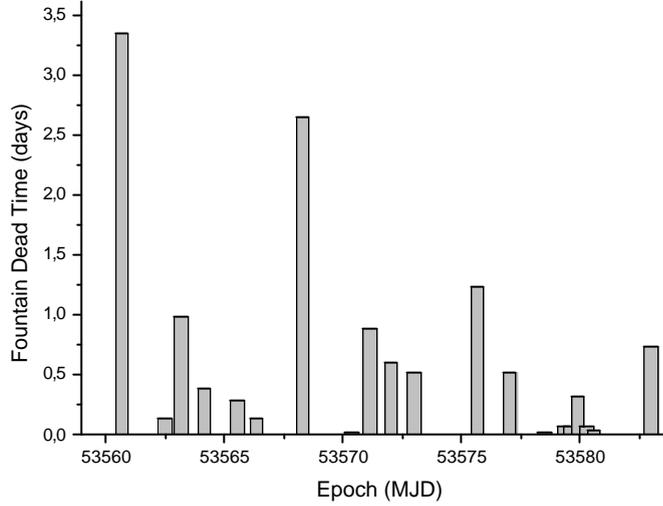


Figure 2. Epoch distribution of the dead time during the present evaluation.

$y(IEN-CsF1)-y(HM2)$ data are fitted with the linear model:

$$Y = At + B \quad (1)$$

The estimation of the average frequency $y(IEN-CsF1)-y(HM2)$ during the evaluation interval is $Y|_{t=t_0}$ where t_0 is the evaluation period center (MJD 53571.5 in this particular case). If the epoch coordinate origin is taken on the center of the evaluation interval, the coefficient B, as it is estimated by the weighted least square algorithm, corresponds to the estimation of the average frequency $y(IEN-CsF1)-y(HM2)$ during the evaluation interval.

The linear fit is weighted on the squared Type A uncertainty of each $y(IEN-CsF1)-y(HM2)$ datum. The uncertainty of each datum includes both the uncertainty due to the fountain stability and the uncertainty due to the collision shift evaluation (Type A contribution). The uncertainty associated to the average frequency estimation $y(IEN-CsF1)-y(HM2)$ and reported as Type A uncertainty is the uncertainty of the coefficient B as it is estimated by the weighted least square algorithm. Figure 3 reports $y(IEN-CsF1)-y(HM2)$ data, corrected for the total shift reported in Table 1, and the linear fit curve.

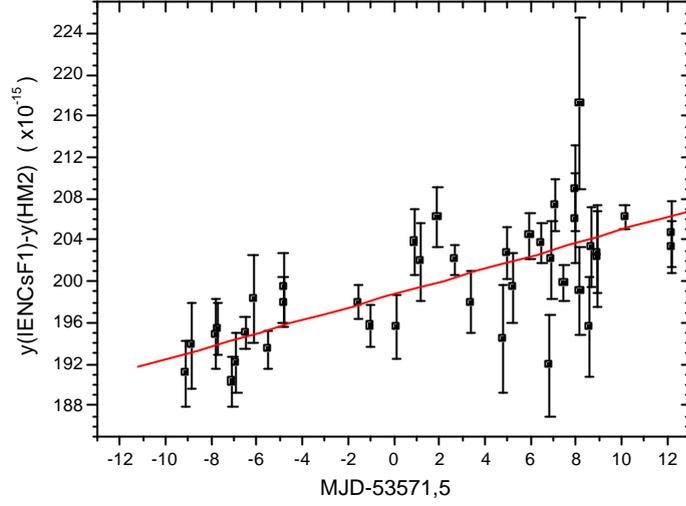


Figure 3. Shift corrected $y(IEN-CsF1)-y(HM2)$ data (squares) and the linear fit curve (straight line).

The linear regression provides the best estimation when the expression (1) is the correct model for the maser drift and the fit residuals are dominated by white frequency noise. As no high stability local oscillator other than HM2 was running at IEN during fountain evaluation period, it is difficult to prove the two positions reported above. However, with the help of all the data collected during the past fountain evaluations and the operative life of HM2 [9], one can reasonably assess that, for a 25 days long period, the fit residuals are dominated by the white frequency noise of the fountain and higher order drifts of the maser are negligible. Final results of the statistical analysis is reported in Table 2:

	Value	Uncertainty
Coefficient A	$0.63 \cdot 10^{-15}$ /day	$0.06 \cdot 10^{-15}$ /day
Coefficient B	$+198.7 \cdot 10^{-15}$	$0.4 \cdot 10^{-15}$

Table 2. Results of the weighted linear fit $y=At+B$.

Local link and dead time uncertainty (ul/lab)

The HM2 is phase compared to UTC(IEN) time scale, which is the reference time scale for remote time and frequency transfer tools, with a Time Interval Counter in the IEN Time and Frequency laboratory. This comparison introduces a uncertainty contribution to the IEN-CsF1 transfer to TAI, which is estimated as $0.1 \cdot 10^{-15}$ for this evaluation period (25 days).

Dead time in fountain operation introduces a further uncertainty to the frequency transfer to TAI. The estimation of this uncertainty contribution requires the knowledge of the HM2 noise properties.

A conservative estimation is possible using, for example, the stability analysis of the $y(IEN-CsF1)-y(HM2)$ data obtained during the fountain comparison experiment in 2004 [9]. This analysis provides that the stability of HM2 could be modelled in terms of Allan variance, as:

$$\mathbf{s}_y^2(\mathbf{t}) = \mathbf{s}_{yWF}^2(\mathbf{t}) + \mathbf{s}_{yFF}^2(\mathbf{t}) + \mathbf{s}_{yRW}^2(\mathbf{t})$$

where $\sigma_{yWF}^2(\tau)$, $\sigma_{yFF}^2(\tau)$ and $\sigma_{yRW}^2(\tau)$ are respectively the contribution due to white, flicker and random walk frequency noise.

A conservative estimation of these contributions is:

$$\begin{aligned} \mathbf{s}_{yWF}(\mathbf{t}) &= 3 \cdot 10^{-13} \mathbf{t}^{-1/2} \\ \mathbf{s}_{yFF}(\mathbf{t}) &< 3 \cdot 10^{-16} \\ \mathbf{s}_{yRW}(\mathbf{t}) &< 2 \cdot 10^{-19} \mathbf{t}^{1/2} \end{aligned} \quad (3)$$

The dead time uncertainty contribution is calculated with the following formulas [8,10]:

$$\begin{aligned} \mathbf{s}_{dWF}(\Delta T) &= \frac{\mathbf{s}_{yWF}(1s)}{\sqrt{\Delta T}} \sqrt{x} \\ \mathbf{s}_{dFF} &\approx \sqrt{B_2 B_3 - 1} \mathbf{s}_{yFF} \\ \mathbf{s}_{dRW}(\Delta T) &\approx \sqrt{[(1-x)B_2 B_3 - 1]} \mathbf{s}_{yRW}(1s) \sqrt{\Delta T} \end{aligned} \quad (4)$$

where σ_{dWF} , σ_{dFF} and σ_{dRW} are the contribution to the dead time uncertainty due to white, flicker and random walk frequency noise of the local oscillator; DT is the evaluation period, $\mathbf{s}(1s)$ is the stability of the local oscillator at 1s, x is the fractional dead-time, B_2 and B_3 are the bias functions defined in [11]. B_2 and B_3 values depends on the noise type (flicker or random walk); B_3 value depends also on the temporal distribution of the dead time (see Figure 2). As the dead time is not regularly distributed, for the B_3 calculation we considered the dead time as lumped, which implies a conservative estimation of the uncertainty.

In the present evaluation run, the dead time amount was 52% of the total measurement; the three terms $\sigma_{dWF}(\tau)$, $\sigma_{dFF}(\tau)$ and $\sigma_{dRW}(\tau)$ have been calculated and the total uncertainty due to the dead time is evaluated to be $\sigma_d = 3 \cdot 10^{-16}$.

Contribution	Uncertainty (10^{-15})
HM link to UTC(IEN)	0.1
Fountain Dead Time (52 %)	0.3
Total (ul/lab)	0.3

Table 3. Contributions to ul/lab.

Summary of TAI evaluation results

MJD Period	y(IENCsF1-HM2)	uA	uB	ul/lab
53559-53584	+198.7 · 10 ⁻¹⁵ (*)	0.4 · 10 ⁻¹⁵ (**)	0.9 · 10 ⁻¹⁵	0.3 · 10 ⁻¹⁵ (***)

Table 4. Final results of IEN-CsF1 evaluation.

(*) HM2 has the BIPM code 1401102

(**) Including collisional shift evaluation uncertainty (Type A contribution)

(***) Including contribution of uncertainties due to the local link to UTC(IEN) and to the fountain dead time.

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