

**FREQUENCY COMPARISON (H_MASER 40 0805) - (UTC(OP))
From MJD 53514 to MJD 53534**

The hydrogen Maser (40 0805) of LNE-SYRTE Time & Frequency laboratory was compared to the UTC(OP) time scale of Paris Observatory, from MJD 53514 to MJD 53534.

The mean frequency differences measured between the hydrogen Maser 40 0805 and UTC(OP) time scale during this period is given in table 1. Figure 1 shows the phase differences measurements recorded during this period.

Period (MJD)	$y(\text{HMaser}_{40\ 0805} - \text{UTC(OP)})$ (I)	$u_{\text{link / maser}}$ (III)
53514 – 53534	+ 587,0	0,16

Table 1: Results of the comparison in 1×10^{-15} unit.

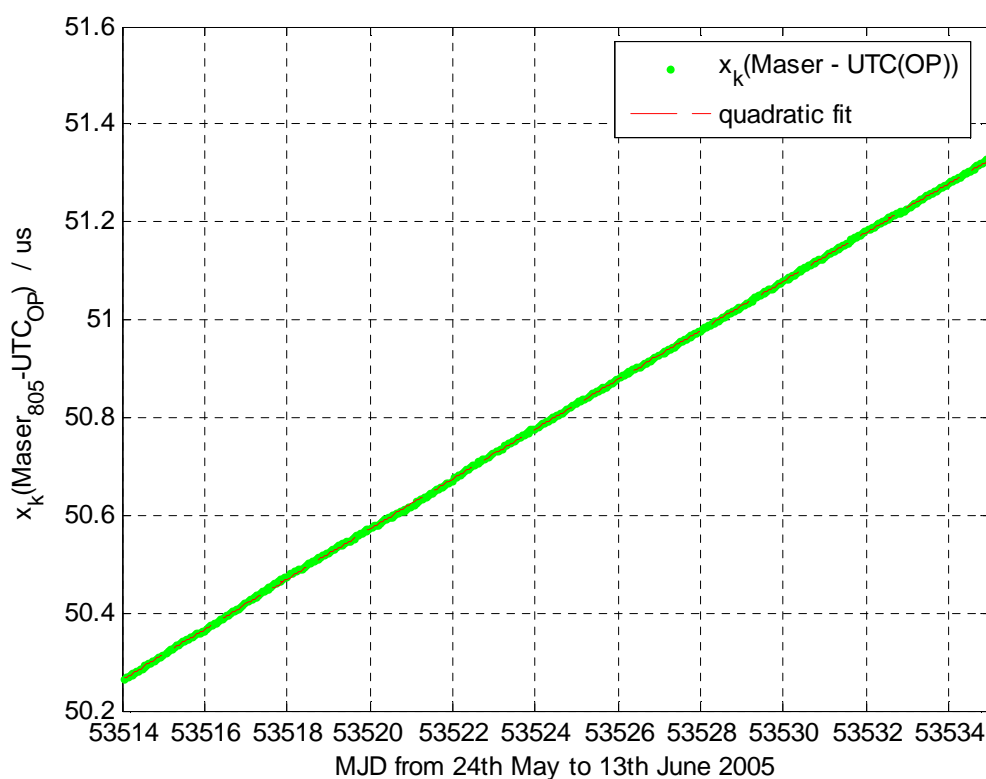


Figure 1: phase differences measurements between Maser 40 0805 & UTC(OP) every one hour

I- Frequency average between hydrogen maser and UTC(OP)

- We search a polynomial interpolation of order 2 written as $x(t) := p_1 t^2 + p_2 t + p_3$ where $x(t)$ represents the phase differences between Maser and UTC(OP) with t a normalized time

$$t = \frac{t_{MJD} - m}{s}, \quad m = \frac{\sum_{i=1}^n t_i}{n} \text{ and } s = \sqrt{\frac{\sum_{i=1}^n (t_i - m)^2}{n}}.$$

The frequency mean is evaluated by the

classical formula

$$y_{moy} = \frac{1}{86400} \left(\frac{1}{MJD_{fin} - MJD_{deb}} \int_{MJD_{deb}}^{MJD_{fin}} \frac{d}{dt} x(t) dt \right) \text{ that gives:}$$

$$y_{moy} = \frac{1}{86400} \frac{(MJD_{fin} - 2m + MJD_{deb}) p_1}{s^2} + \frac{1}{86400} \frac{p_2}{s}$$

The frequency mean evaluated by the quadratic fit from MJD 53514 to MJD 53534 is then

$$y_{H805-UTC(OP)} = 0.587057 \cdot 10^{-12}$$

- by using a linear fit $x(t) := p_1 t + p_2$ on phase differences with t a normalized time $t = \frac{t_{MJD} - m}{s}$,

$$m = \frac{\sum_{i=1}^n t_i}{n} \text{ and } s = \sqrt{\frac{\sum_{i=1}^n (t_i - m)^2}{n}} \text{ we obtain a frequency average}$$

$$y_{moy} = \frac{1}{86400} \left(\frac{1}{MJD_{fin} - MJD_{deb}} \int_{MJD_{deb}}^{MJD_{fin}} \frac{p_1}{s} dt \right) = \frac{1}{86400} \frac{p_1}{s}$$

$$y_{H805-UTC(OP)} = + 587,065 \times 10^{-15}$$

- by using the average of first phase differences $y_{moy} = \frac{\sum_{k=1}^N y_k}{N}$ with $y_k = \frac{x(t_{k+1}) - x(t_k)}{t_{k+1} - t_k}$ we obtain

$$y_{H805-UTC(OP)} = + 588,483 \times 10^{-15}$$

- by using the phase difference between the ending and the beginning of this MJD period

$$y_{moy} = \frac{1}{86400} \frac{x_{fin} - x_{deb}}{MJD_{fin} - MJD_{deb}} \text{ it gives}$$

$$y_{H805-UTC(OP)} = + 589,14 \times 10^{-15}$$

II- Stability analyses

The quadratic fit was removed from data measurements and residues were analyzed and plotted on figure 2. A span of ± 5 ns is exhibited during the 20 days period of measurement. The frequency stability analysis using these residues is plotted in figure 3 with the Modified Allan Deviation computation.

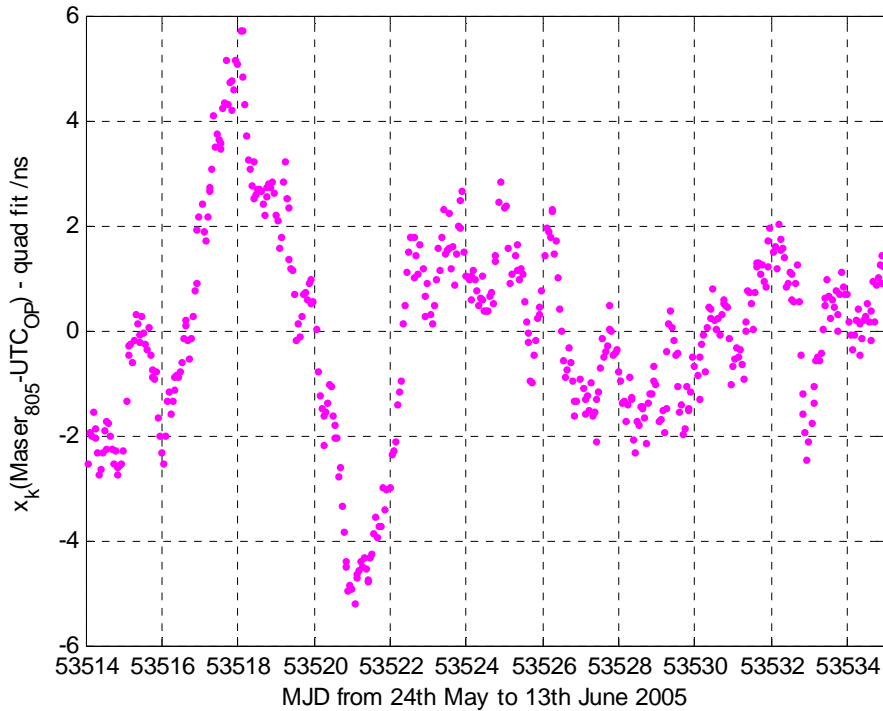


Figure 2: phase differences Maser 40 0805 & UTC(OP) quad fit removed

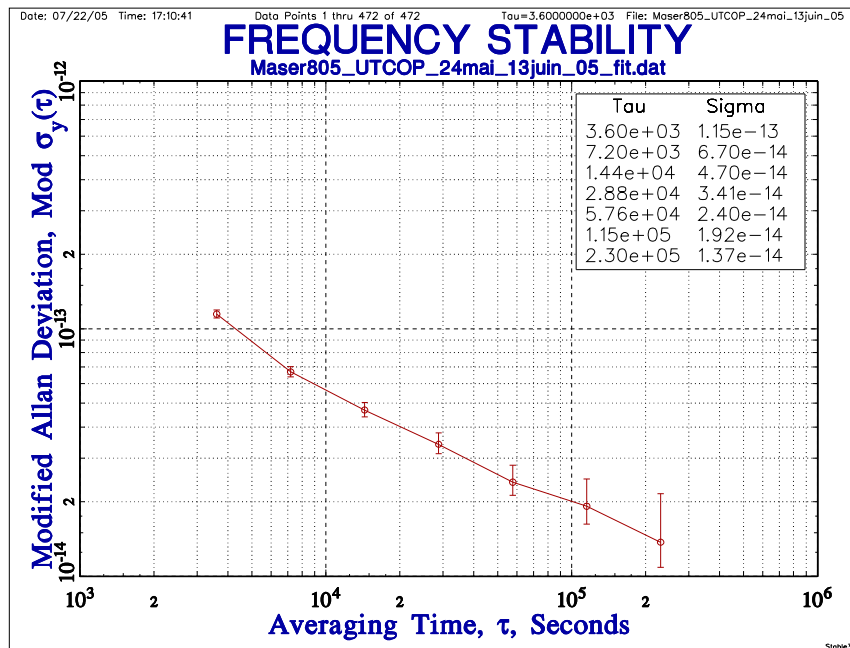


Figure 3: Frequency stability between Maser 40 0805 & UTC(OP)

If we take the Allan deviation extrapolated at 20 days following a white frequency noise of law $\tau^{-1/2}$ that gives an evaluation of frequency stability:

$$\sigma_A(\tau=20d) = 5,2 \times 10^{-15}$$

This noise doesn't reflect the link noise between Maser and UTC(OP). It gives the principal noise contribution of the atomic Caesium clock Agilent 5071A.

III- Uncertainty evaluation of the link Maser-UTC(OP)

Phase differences are measured with a time interval counter HP5370B with a resolution of 200ps by sample.

An estimation of link noise between Maser and UTC(OP) is given by taking the first and the end of phase differences measurement with 200ps uncertainty each. The frequency difference Maser – UTC(OP) have an uncertainty evaluated by the rate

$$y_{moy} = \frac{x_{fin} - x_{deb}}{86400 MJD_{fin} - 86400 MJD_{deb}}$$

And considering $|\delta(x_{deb})| = |\delta(x_{fin})|$ we find

$$\delta(y_{moy}) = \frac{1}{86400} \frac{\sqrt{2} \sqrt{\delta(x_{deb})^2}}{MJD_{fin} - MJD_{deb}}$$

With $\delta(x) = 0.200 \cdot 10^{-9}$ second that gives:

$$\delta(y_{moy}) = 0.157 \cdot 10^{-15}$$