FREQUENCY COMPARISON (H_MASER 140 0810) - (LNE-SYRTE-FO2)
For the period MJD 58199 to MJD 58234

The primary frequency standard LNE-SYRTE-FO2 has been compared to the hydrogen Maser 140 0810 of the laboratory, during a measurement campaign between MJD 58199 and 58234 (22nd March 2018 – 26th April 2018). The hydrogen maser presented a frequency step around the middle of the measurement period, between MJD 58214 and 58219. Therefore this period was not used for the maser calibration, and the collected data have been separated into two intervals of 15 days, respectively. The fountain operation covered 99.1% and 95.6% of the two periods, respectively.

The mean frequency differences at the middle date of each interval are given in the following table:

Period (MJD)	Date of the estimation	y(HMaser$_{140\ 0810}$ – FO2)	u_B	u_A	$u_{link\ /\ maser}$
58199 – 58214	**58206.5**	**-6046.3**	**2.0**	**2.0**	**0.5**
58219 – 58234	**58226.5**	**-6151.3**	**2.0**	**2.0**	**0.5**

Table 1: Results of the comparison in 1 × 10^{-16}.

The FO2 fountain was operated in the same mode during all the period: the interrogating signal is based on the down conversion to 9.192 GHz of a 11.98 GHz signal provided by a cryogenic oscillator phase locked to the maser 140 0810. A synthesizer is used to lock the microwave signal to the atomic resonance. The frequency difference between this maser and the fountain is deduced from the average correction applied to the synthesizer.

Average value and statistical uncertainty

The frequency data are averaged over 0.2 day intervals. We then perform a linear unweighted fit to the average data points to determine the average frequency at the middle date of the interval, as given in Table 1. The statistical uncertainty u_A is estimated using the Allan variance of the frequency residuals, after removing the drift. We estimate conservative statistical uncertainties u_A of 2.0 × 10^{-16} for the two periods, respectively.

We verified the result by applying a second method. We calculated the accumulated phase by integrating the data points, assuming a constant frequency during each segment, and during the dead times of the fountain operation. The average frequency is then obtained by dividing the total accumulated phase by the calibration period duration. The processing has been performed with segments of 0.01, 0.1 and 1 day durations. The results are in agreement with the values given in Table 1 within 0.2 × 10^{-16} and 1.4 × 10^{-16} for the two segments, respectively, which is consistent with the estimation of the statistical uncertainties u_A and the uncertainties due to the link.

Accuracy

The frequency is corrected from the quadratic Zeeman, the Black Body radiation, the cold collisions and cavity pulling, the first order Doppler, the microwave lensing, and the redshift effects. Here the uncertainty in the cold collisions correction accounts for both a statistical uncertainty and a systematic uncertainty taken as 0.5% of the correction for high density measurements. The following table summarizes the budget of systematic effects and their associated uncertainties. The accuracy is the quadratic sum of all the systematic uncertainties.

	Correction (10^{-16})	Uncertainty (10^{-16})
Quadratic Zeeman effect	-1935.29	0.30
Black body radiation	171.14	0.60
Cold collisions and cavity pulling	137.79	0.87
Distributed cavity phase shift	-0.90	1.00
Microwave spectral purity&leakage	0	< 0.50
Ramsey & Rabi pulling	0	< 0.10
Microwave lensing	-0.70	0.70
Second order Doppler effect	0	< 0.10
Background gas collisions	0	<1.00
Total	-1627.96	1.99
Redshift (*)	- 65.54	0.25
Total with redshift	**-1693.50**	**2.01**

Table 2: Budget of systematic effects and uncertainties for SYRTE-FO2 fountain for the MJD 58199 – 58234 period

$$\boxed{u_B = \mathbf{2.0 \times 10^{-16}}}$$

(*) From now on for the LNE-SYRTE fountains, we use an improved relativistic redshift corrections with reduced uncertainties. Within the ITOC (International Timescales with Optical Clocks) project, in order to calculate the relativistic redshift corrections for the clocks hosted at the four European metrology laboratories INRIM, LNE-SYRTE, NPL, and PTB, the gravity potentials at local reference markers at each site were newly determined with respect to a common reference potential [1,2]. This involved a combination of GNSS based height measurements, geometric levelling and a European geoid model, refined by local gravity measurements. Two well-known geodetic methods, GNSS/Geoid and geometric levelling, were used in order to estimate the geopotential numbers C with respect to the common reference potential: We take the value C(GNSS/geoid) based on the GNSS/Geoid method, which is more accurate in the context of realizing international timescales. The difference ($\Delta C = 0.109$ m^2/s^2) between the results of both methods can be taken as an estimate of the combined uncertainty of both methods. We take twice this value as a conservative uncertainty for the corresponding redshift correction C(GNSS/geoid)/c^2, i.e. 2.4×10^{-17} for LNE-SYRTE clocks/fountains. From Table 6 in ITOC project deliverable D4.2, the C(GNSS/geoid) number for the local reference marker for FO2 is 57.958×10 m^2/s^2. Next, from the fountain geometry and atom cloud launch velocity, we determine the height of the FO2-Cs average atomic trajectories above the reference marker, 0.962 m with 0.01 m uncertainty. Then, using the local g value, we get the relativistic redshift

$$\text{FO2-Cs redshift} = (57.958 \times 10\ m^2/s^2 + 0.962\ m \times 9.809276476\ m/s^2)/c^2 = 65.537 \times 10^{-16}$$

We take an overall conservative uncertainty of 2.5×10^{-17} on the corresponding redshift correction.

[1] H. Denker et al., J Geod (2018) 92:487–516: "Geodetic methods to determine the relativistic redshift at the level of 10^{-18} in the context of international timescales: a review and practical results".
[2] P. Delva, H. Denker and G. Lion, arXiv:1804.09506: "Chronometric geodesy: methods and applications"

Uncertainty of the link

The uncertainty of the link is the quadratic sum of 2 terms:
-A possible effect of phase fluctuations introduced by the cables that connect the primary standard to the maser. A new characterization of the signal distribution leads to a still conservative value of 0.5×10^{-16}.
-The uncertainty due to the dead times of the frequency comparison.
We have updated the estimation of this contribution, applying the method described in *Metrologia*, vol. 44, pp 91-96, 2007, as we did for the initial calibration reports of the LNE-SYRTE Strontium SFS. The maser noise model includes a white frequency noise component of 5×10^{-16} at 1 d and a flicker frequency noise component of 5×10^{-16} at 1 d, which is pessimistic especially for short averaging periods. We applied the method to the dead times longer than 600 s and obtained a stability degradation of 0.1×10^{-16} and 0.2×10^{-16} for the two segments, respectively.