

Evaluation of PTB primary caesium fountain frequency standard CSF1 between MJD 56199 - MJD 56214

PTB's primary caesium fountain frequency standard CSF1 was operated between MJD 56199, 0:00 UTC and MJD 56214, 0:00 UTC. Frequency comparisons were made with respect to PTB hydrogen maser H5, BIPM code 1400590, using a 5 MHz phase comparator.

The relative frequency instability of CSF1 was $1.49 \times 10^{-13} \cdot (\tau/s)^{-1/2}$ during the 15 days. This results in a statistical uncertainty of 0.13×10^{-15} assuming that white frequency noise is the dominant noise source. Because the frequency comparison over the 15 days is made with an uncertainty - due to the instrumentation - of below 0.22×10^{-15} the resulting total statistical uncertainty is given by $u_A (\tau = 15 \text{ d}) = 0.26 \times 10^{-15}$.

In total 12887 comparison data points for intervals of 100 s duration were obtained, corresponding to 99.44% of the 15×24 hours. From these numbers an uncertainty due to the clock link $u_{\text{Lab}} < 0.02 \times 10^{-15}$ is obtained. The estimated uncertainty for the link to TAI for 15 days is $u_{\text{TAI}} = 0.12 \times 10^{-15}$.

Frequency corrections for the following effects were applied to the raw data:

- Zeeman effect (magnetic field along the atoms' trajectory),
- black body effect (thermal radiation along the atoms' trajectory),
- cold collisions effect,
- gravitational red-shift and relativistic Doppler effect.

The CSF1 standard uncertainty u_B is estimated as 0.74×10^{-15} (1σ) for the relevant period.

Table of results of CSF1 compared to hydrogen maser H5 (1400590)

Interval of evaluation	MJD 56199, 0:00 UTC - MJD 56214, 0:00 UTC
Fractional dead time	0.56%
Resulting frequency difference	$y(\text{CSF1} - \text{H5}) = 70.80 \times 10^{-15}$
Type A uncertainty u_A (1σ)	0.26×10^{-15}
Type B uncertainty u_B (1σ)	0.74×10^{-15}
Link to clock u_{Lab} (1σ)	0.02×10^{-15}
Link to TAI u_{TAI} (1σ)	0.12×10^{-15} (15 days)
Combined uncertainty (1σ)	0.79×10^{-15}

Type A (statistical) uncertainty of CSF1

For the TAI scale unit measurement at hand the frequency instability of CSF1 was measured to be $1.49 \times 10^{-13} (\tau/s)^{-1/2}$. Previously, it had been demonstrated that the employed synthesis setup is capable of providing instabilities below the 10^{-16} level [1]. Using CSF1 for a measurement of the single ytterbium ion clock transition frequency, where the Allan standard deviation was dominated by the white frequency noise of CSF1, a $\tau^{-1/2}$ -dependence down to 4×10^{-16} at 100000 s averaging time could be demonstrated some time ago.

For these reasons the statistical uncertainty of the current TAI scale unit measurement was calculated with the assumption of white frequency noise for the total measurement interval and taking into account a 0.22×10^{-15} uncertainty contribution due to the measurement instrumentation. The resulting total statistical uncertainty is calculated to be $u_A (\tau = 15 \text{ d}) = 0.26 \times 10^{-15}$.

Type B (systematic) uncertainty of CSF1

A detailed description of the PTB fountain CSF1 is given in Refs. [2] and [3]. Below we report some type B uncertainty contributions, which are now treated in a different way or were newly addressed since the last publication of the CSF1 uncertainty budget [3]. The main changes are a more realistic estimate of the frequency uncertainty contribution due to the effect of distributed cavity phase, issue (1), and an additional uncertainty contribution, issue (8), that currently has to be taken into account because of the observation of a frequency shift when CSF1 is operated at elevated microwave power levels.

1) Distributed cavity phase

For the uncertainty evaluation of the second PTB fountain CSF2 [4] the evaluation of the uncertainty contribution due to the effect of distributed cavity phase has been repeated according to the former evaluation of this effect for CSF1 [2]. The main difference is that instead of the formerly used worst case estimates for the trajectories of the atoms, more realistic scenarios have been implemented by using Monte-Carlo simulations for the atomic trajectories. Using the parameters of CSF1, equivalent simulations result in an uncertainty contribution due to the effect of distributed cavity phase of less than 0.1×10^{-15} .

2) Black body radiation shift

For calculating the frequency shifting effect due to the electric field of the ambient temperature radiation we use the results of the most recent evaluation [5]. The outcome for the corresponding frequency shift $\delta\nu_{BB}$ confirms most of the former results and amounts to

$$\delta\nu_{BB} = k_0 E_{300}^2 \left(\frac{T}{300 \text{ K}} \right)^4 \left(1 + \varepsilon \left(\frac{T}{300 \text{ K}} \right)^2 \right)$$

with the ambient temperature T , $E_{300} = 831.9 \text{ V/m}$, and the coefficients $k_0 = -2.282(4) \times 10^{-10} \text{ Hz/(V/m)}^2$ and $\varepsilon = 0.013$.

During the reported 15 days time interval the observed temperature gradients along the vacuum tube of CSF1 were within the uncertainty of the employed PT100 resistances (0.11 K). At the same time the temperature indicated by each single PT100 resistance remained the same within the limits of $\pm 0.2 \text{ K}$. However, for the AC Stark shift correction the calculated average temperature is used. Based on these findings we assume an uncertainty of this temperature of 0.2 K, which gives an uncertainty contribution of $< 0.1 \times 10^{-15}$, also taking into account the given uncertainty of k_0 .

3) Collisional shift

The collisional frequency shift of CSF1 was evaluated a number of times between 2010 and 2012 with otherwise unchanged parameters. For this purpose, the number of atoms contributing to the signal – and in this way the density – was changed by changing the microwave amplitude in the state selection cavity. An electronics switches automatically between two microwave amplitudes of the state selection cavity every 1000 s coherently with the data taking of the 5 MHz phase comparator. In this way a differential measurement of the collisional shift is performed, eliminating the effect of the frequency drifts of the hydrogen maser reference. The results of these evaluations are slope factors which give – multiplied with the actual number of atoms – the collisional frequency shift correction [2], [3].

The three most recent slope factor evaluations agree with the result of other slope factor evaluations in 2010 and 2011 within the statistical uncertainties. For this measurement the average slope factor obtained from these three most recent evaluations has been utilized.

As described in Refs. [2] and [3] the uncertainty of the collisional shift correction is composed of the statistical uncertainty and a 10% systematic uncertainty because of a potentially imperfect proportionality between the measured actual number of atoms and the effective density.

4) Gravitational red shift

The “Institut für Geodäsie und Photogrammetrie” of the Technical University of Braunschweig has newly determined the height above the geoid of a reference point inside PTB’s clock hall. As a result, the gravitational red shift correction has changed by 0.1×10^{-15} with respect to the value given in [2] and has a reduced uncertainty of well below 0.1×10^{-15} .

5) Majorana transitions

In November 2004 during a frequency comparison campaign [6] between several European fountain clocks relative frequency variations of CSF1 of the order of 10^{-14} became apparent. These frequency variations could be traced back to Majorana transitions caused by unintended changes of the properties of the magnetic shield [7]. By proper current settings of correction coils located close to the lower shield caps the

problem could be remedied. The related extensive investigations led to a deeper understanding of the effect of Majorana transitions in fountain clocks and hence enabled us to take further measures in order to avoid such transitions in CSF1. In particular the spatial structure of the magnetic field below the magnetic shield surrounding the C-field region is now better characterised and controlled. Therefore our former estimate of the uncertainty contribution due to Majorana transitions ($< 0.1 \times 10^{-15}$) could be recovered.

6) Microwave leakage

Extended investigations of the effect of microwave leakage in CSF1 were performed [8]. The related results together with the absence of potential leakage fields at the level of -153 dBm lead to an uncertainty contribution due to potential microwave leakage fields of $< 0.1 \times 10^{-15}$.

7) AC Stark shift (light shift)

Extended investigations of a possible frequency shift due to the interaction of the atoms during their ballistic flight with residual light from one of the laser beams used for cooling or detection resulted in a reduced uncertainty estimate of this effect of $< 0.1 \times 10^{-15}$. For these investigations several mechanical shutters were put out of action with the result that no relative frequency shift at the low 10^{-15} level was observed.

8) Microwave power dependence: $1\pi/2$ -pulses, $3\pi/2$ -pulses

It has become a standard procedure to test primary fountain frequency standards at elevated microwave powers in order to check for potential frequency shifts. Usually such tests are performed at odd multiples of $\pi/2$ microwave pulse area, where the microwave power is adjusted to these values typically by maximising the contrast of the central Ramsey fringe. During the past years we investigated shifts of the CSF1 output frequency – exceeding the formerly stated type B uncertainty – that occurred when the main cavity was operated at increased microwave power. Many possible sources of this effect were investigated, with some concentration on Majorana transitions [7] and microwave leakage [8], and could be excluded.

It has been pointed out that under certain conditions the overall collisional shift significantly depends on the clock state composition after the first interaction with the microwave field in the Ramsey cavity, i.e., the ratio of the atom numbers in the clock states ($F = 4, m_F = 0$) and ($F = 3, m_F = 0$) [9]. Notably in the case of a small initial atom cloud size associated with the use of a magneto-optical trap (like in PTB-CSF1), the expansion of the cloud during its ballistic flight results in position-momentum correlations, which in turn lead to decreasing relative atom velocities in a collision. At these low collisional energies the collisional cross-section is markedly different for atoms in the two clock states [9]. The overall collisional shift therefore depends on the composition of the atomic state after the first Ramsey interaction.

It has been experimentally demonstrated that in this case for different multiples of $\pi/2$ pulse area a varying clock state composition after the first Ramsey interaction can be obtained, if the average microwave power seen by the expanding atom cloud is different during the first and the second transition through the Ramsey cavity [10]. In the

presence of a clock state composition dependent collisional shift, this effect gives rise to different collisional shifts for operation at different multiples of $\pi/2$ pulse area.

Taking this into account we are able to explain the major part – if not all – of the peculiarities found in PTB-CSF1 at multiple $\pi/2$ pulse area operation, as confirmed by experiment. The currently observed remaining frequency difference for operation at $1\pi/2$ pulse area and $3\pi/2$ -pulse area is $(1.1 \pm 0.5) \times 10^{-15}$. In order to take this difference into account we have added an uncertainty contribution of half of the measured frequency difference (i.e., 0.6×10^{-15}) to the uncertainty budget for the normal mode of operation at $1\pi/2$ pulse area.

Frequency shifts, corrections and type B uncertainties of CSF1 (parts in 10^{15}):

Frequency shift	Correction	Uncertainty
Quadratic Zeeman shift	- 46.20	0.10
Black body radiation shift	16.53	0.10
Gravity+relativistic Doppler effect	- 8.58	0.10
Collisional shift	1.10	0.23
Cavity phase shift		0.10
AC Stark shift (light shift)		0.10
Majorana transitions		0.10
Rabi and Ramsey pulling		0.10
Electronics		0.20
Microwave leakage		0.10
Background gas collisions		0.10
Microwave power dependence		0.60
Total type B uncertainty		0.74

References

- [1] A. Sen Gupta, R. Schröder, S. Weyers and R. Wynands, 21st European Frequency and Time Forum (EFTF), Geneva, pp. 234–237 (May/June 2007)
- [2] S. Weyers, U. Hübner, R. Schröder, Chr. Tamm, A. Bauch, *Metrologia* **38** (4), pp. 343–352 (2001)
- [3] S. Weyers, A. Bauch, R. Schröder, Chr. Tamm, in: Proceedings of the 6th Symposium on Frequency Standards and Metrology 2001, University of St Andrews, Fife, Scotland, pp. 64–71, ISBN 981-02-4911-X (World Scientific)
- [4] V. Gerginov, N. Nemitz, S. Weyers, R. Schröder, D. Griebisch, R. Wynands, *Metrologia*, **47**(1), 65-79 (2010)
- [5] P. Rosenbusch, S. Zhang, and A. Clairon, 21th European Frequency and Time Forum (EFTF), Geneva, pp. 1060–1063 (June 2007)
- [6] A. Bauch, J. Achkar, S. Bize, D. Calonico, R. Dach, R. Hlavac, L. Lorini, T. Parker, G. Petit, D. Piester, K. Szymaniec and P. Urich, *Metrologia* **43**, pp. 109–120 (2006)
- [7] S. Weyers, R. Schröder, R. Wynands, 20th European Frequency and Time Forum (EFTF), Braunschweig, pp. 219–223 (March 2006)
- [8] S. Weyers, R. Schröder, R. Wynands, 20th European Frequency and Time Forum (EFTF), Braunschweig, pp. 173–180 (March 2006)
- [9] K. Szymaniec, W. Chalupczak, E. Tiesinga, C. J. Williams, S. Weyers, R. Wynands, *Physical Review Letters* **98**, 153002 (April 2007)
- [10] K. Szymaniec, W. Chalupczak, S. Weyers, R. Wynands, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **54**(9), pp. 1721–1722 (September 2007)