

# Frequency evaluation of the NIM6 PFS against H-maser 1404821 for the period from MJD 60519 to MJD 60549

## 1. Summary

During the period from MJD 60519 to MJD 60549 (28<sup>th</sup> July 2024–26<sup>th</sup> August 2024), the NIM Primary Frequency Standard NIM6 [1] was used to measure the fractional frequency offset of the NIM H-maser (BIPM code 1404821). During the measurement, NIM6 operated alternatively between high and low-density modes. Table 1 below summarizes the evaluation result as well as the associated uncertainties.

Table 1. Results of NIM6 compared to H-maser 1404821.

|                                                            |                         |
|------------------------------------------------------------|-------------------------|
| Measurement period                                         | MJD 60519-MJD 60549     |
| Reference clock (H-maser)                                  | BIPM code 1404821       |
| Uptime ratio                                               | 99.1%                   |
| Frequency difference $y$ (NIM6-HMaser <sub>1404821</sub> ) | $119.1 \times 10^{-16}$ |
| Type-A uncertainty $u_A$                                   | $2.3 \times 10^{-16}$   |
| Type-B uncertainty $u_B$                                   | $2.3 \times 10^{-16}$   |
| Link to H-maser $u_{A/lab}$                                | $< 1.0 \times 10^{-16}$ |
| Link to H-maser $u_{B/lab}$                                | $< 0.1 \times 10^{-16}$ |

The final uncertainty into TAI  $u_{total}$  is the square sum of the five uncertainties as following:

$$u_{total} = \sqrt{(u_A)^2 + (u_B)^2 + (u_{A/lab})^2 + (u_{B/lab})^2 + (u_{link/TAI})^2} \quad (1)$$

where  $u_A$  is the statistical uncertainty on the frequency measurement, and  $u_B$  is the systematic uncertainty of the NIM6 [1]. The link uncertainty  $u_{A/lab}$  comprises two terms and is determined through the quadratic summation of these two terms [1].  $u_{B/Lab}$  represents the systematic uncertainty of the frequency link. The 100 MHz reference signal from H-maser 1404821 is directly connected to the microwave synthesizer of NIM6 via a long cable. This signal is looped back from NIM6's laboratory to the lab housing H-maser 1404821, where the frequency difference and stability are monitored. No frequency difference has been observed within the measured stability limits. Therefore, the associated type B uncertainty due to the link  $u_{B/lab}$  is estimated to be less than  $1.0 \times 10^{-17}$ . The frequency transfer uncertainty  $u_{link/TAI}$  arises from H-maser 1404821 to TAI through the remote frequency comparison link. All the above uncertainties are calculated at  $1\sigma$ .

## 2. Systematic frequency corrections and uncertainties

The following frequency corrections are applied to the raw NIM6-HMaser<sub>1404821</sub> data, and the corresponding systematic frequency uncertainties are incorporated into the fountain fractional uncertainty budget. The values are listed in Table 2.

—— *Second-order Zeeman shift*

—— *Cold collisions shift*

The cold collisions shift [1] was estimated during the measurement by operating alternatively between high and low atomic densities (or atom number) through adjusting the power of the state selection microwave pulse. According to the measured relative frequencies, along with the detected atomic numbers at the high and low densities, the relative frequency extrapolated to zero density is given by

$$f_{\text{zero}} = \frac{kf_L - f_H}{k - 1} \quad (2)$$

where  $k$  is the ratio ( $N_H/N_L$ ) between high and low densities, and  $f_H$  and  $f_L$  are the measured frequencies at high and low densities respectively. At low and high density, the collisional shifts are given by  $(f_H - f_L)/(k-1)$  and  $k(f_H - f_L)/(k-1)$ , respectively.

The uncertainty in eliminating the collision frequency shift is derived from equation (2) as

$$\sigma_z^2(\tau) = \left(\frac{k}{k-1}\right)^2 \sigma_L^2(\tau/2) + \left(\frac{1}{k-1}\right)^2 \sigma_H^2(\tau/2) + \left(\frac{f_L - f_H}{(k-1)^2}\right)^2 \sigma_k^2 \quad (3)$$

here,  $\tau/2$  represents the averaging time for a single density measurement,  $\sigma_k$  is the uncertainty of the ratio  $k$ . The first two terms are treated as type-A uncertainty. The last term is treated as the cold-collision-induced type-B uncertainty and included in the systematic uncertainty budget, which includes the nonlinearity  $\sigma_{\text{nonlinear}}$  between the measured atom numbers and the average density, and the atom number uncertainties  $\delta N_L$ ,  $\delta N_H$ , is expressed as [1]

$$\left\{ \begin{array}{l} \delta\left(\frac{\Delta f}{f_0}\right)_{\text{collision}} = \frac{1}{f_0} \left(\frac{f_H - f_L}{(k-1)^2}\right) \sigma_k \\ \sigma_k^2 = \sigma_{\text{nonlinear}}^2 + \left[ \left(\frac{\delta N_L}{N_L}\right)^2 + \left(\frac{\delta N_H}{N_H}\right)^2 \right] k^2 \end{array} \right. \quad (4)$$

Given a  $k$  of 2.30, a  $(f_L - f_H)/f_0$  of  $2.44 \times 10^{-15}$ , a  $\sigma_{\text{nonlinear}}$  of 0.05 horizontally and 0.01 vertically, and atom number variations of less than 3%, the fractional uncertainty induced by cold collisions is evaluated to be  $1.59 \times 10^{-16}$ .

#### ——**Blackbody radiation shift**

#### ——**Relativistic red and second-order Doppler shifts**

The gravitational red shift [1] was calculated from the reference gravitational potential  $W_0 = 62636856.0 \text{ m}^2\text{s}^{-2}$  for TT/TAI, the gravitational potential  $W_{\text{QB}} = 62636852.95 \pm 0.49 \text{ m}^2\text{s}^{-2}$  at the mean sea level at the tide gauge station in Qingdao of China, the local gravitational acceleration  $g$  and the height of the fountain's Ramsey cavity. For an atomic cloud launched to a height of  $h_{\text{ac}} = 0.336(7) \text{ m}$  above the mid-plane of the Ramsey cavity, the combined fractional frequency shift due to the relativistic (red and second-order Doppler) effects can be expressed as  $\Delta f/f_0 = h_{\text{ac}}g/(3c^2)$  [2]. For NIM6 fountain clock, this shift is evaluated to be  $0.12 \times 10^{-16}$  with a fractional uncertainty of  $3 \times 10^{-19}$ . In summary, the combined fractional frequency shift due to the relativistic red and second-order Doppler shifts is  $86.52 \times 10^{-16}$  with a fractional uncertainty of  $0.2 \times 10^{-16}$ .

#### ——**Microwave lensing shift**

The following systematic frequency uncertainties are added to the NIM6 uncertainty budget. The values are listed in Table 2.

- Microwave interferometric switch*
- Microwave leakage*
- Distributed Cavity Phase (DCP)*
- Microwave spectrum impurity*
- Light shift*
- Majorana transitions*
- Rabi and Ramsey pulling*
- Cavity pulling*
- Collision with background gases*

### 3. Frequency measurement results

The systematic frequency shifts and their uncertainties are listed in Table 2. A more detailed description of the shifts and uncertainties is given in [1].

Table 2. Uncertainty budget of the NIM6, listing physical effects, frequency corrections, and fractional type-B uncertainty in a unit of  $10^{-16}$ .

| Physical effect                                  | Bias / $10^{-16}$ | Uncertainty/ $10^{-16}$ |
|--------------------------------------------------|-------------------|-------------------------|
| Second-order Zeeman                              | 728.8             | 0.7                     |
| Cold collisions                                  | 0.0*              | 1.59                    |
| Blackbody radiation                              | -165.9            | 0.6                     |
| Relativistic red and second-order Doppler shifts | 86.52             | 0.2                     |
| Microwave lensing                                | 0.63              | 0.5                     |
| Microwave interferometric switch                 | 0.0               | 1.0                     |
| Microwave leakage                                | 0.0               | 0.1                     |
| DCP                                              | 0.0               | 0.87                    |
| Microwave spectral impurities                    | 0.0               | 0.1                     |
| Light shift                                      | 0.0               | 0.01                    |
| Majorana transition                              | 0.0               | 0.1                     |
| Rabi and Ramsey pulling                          | 0.0               | 0.1                     |
| Cavity pulling                                   | 0.0               | 0.02                    |
| Collision with background gases                  | 0.0               | 0.1                     |
| <b>Total</b>                                     | <b>650.05</b>     | <b>2.3</b>              |

\*The fractional collisional shift is calculated to be  $18.8 \times 10^{-16}$  at low density and  $43.2 \times 10^{-16}$  at high density.

### References

- [1] F. S. Zheng, W. L. Chen, K. Liu, S. Y. Dai, N. F. Liu, Y. Z. Wang and F. Fang, “Uncertainty Evaluation of the Caesium Fountain Primary Frequency Standard NIM6,” *Metrologia*, vol. 62, no. 3, 035005, 2025.
- [2] V. Gerginov, G. W Hoth, T. P Heavner, T. E Parker, K. Gibble and J.A Sherman, “Accuracy evaluation of primary frequency standard NIST-F4,” *Metrologia*, vol. 62, no. 3, p. 035002, 2025.